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SOME FIXED POINT RESULTS IN FUZZY METRIC SPACES USING A CONTROL FUNCTION

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Abstract. In this paper, we establish the results on existence and uniqueness of fixed point for ϕ-contractive and generalized C-contractive mapping in the fuzzy metric space in the sense of George and Veeramani. We use the notion of altering distance for proving the results.

1 Introduction

Menger [19] introduced an interesting and important generalization of the metric space called probabilistic metric space in 1942. The idea was to use distribution functions instead of non-negative real numbers as values of the metric. Kramosil and Michalek [18] introduced fuzzy metric space as a generalization of Menger spaces. Later George and Veermani [11] modified the notion of fuzzy metric spaces. They imposed some conditions on the fuzzy metric space in order to obtain a Hausdorff topology. In this paper we consider some fixed point problems in the fuzzy metric spaces defined in the sense of George and Veeramani.

Fixed point theory is an active branch of research. Sehgal and Bharucha-Reid [26] introduced the notion of contraction mapping in probabilistic metric spaces. They studied the existence and uniqueness of fixed point for B-contraction on a complete Menger space. Hicks [16] introduced the class of probabilistic C-contractions which was different from Sehgal's contraction. After that fixed point theory in probabilistic and fuzzy metric spaces developed in different directions. A comprehensive survey of research in this line was given by Hadzic and Pap in [14]. Some of recent references probabilistic and fuzzy metric spaces may be noted in [2, 5, 6, 7, 9, 10, 13] and [27].

In 1984 Khan et al [17] introduced the notion altering distance function and using it they had proved some fixed point theorems in complete metric spaces.

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Altering distance function has been used in a number of works in metric fixed point theory. Some of the results are noted in [8, 23] and [24]. The concept of altering distance function has been generalized to two variables and three variables in [1] and [3] respectively. This notion has also been used to prove fixed point results for multivalued and fuzzy mappings in [4].

With a view to extending the idea of altering distance function to probabilistic metric spaces Choudhury and Das [5] introduced a new contraction in Menger spaces. The contraction involves a class of real function, known as Φ-function and generalizes the Sehgal's contraction. Further fixed point results by use of Φ-functions have been established in [6, 7, 9] and [22].

In this paper we prove some fixed point results in fuzzy metric spaces by use of Φ-functions. We use the concept of p-convergence. This type of convergence was introduced by Mihet in [21]. We also support our result by examples.

2 Section

In this section we give some definitions and results which are needed for our discussion.

Definition 1. *[14, 25] A mapping $F : (-\infty, \infty) \to [0, 1]$ is called a distribution function if it is nondecreasing and left continuous on $[0, 1]$ with $F(0) = 0$.*

The class of all distribution functions is denoted by Δ_+.

Definition 2. ***Probabilistic metric Space [14, 25]***
A probabilistic metric space is an order pair (X, F) where X is a nonempty set and F is a mapping from $X \times X$ to Δ_+ (denoted by $F_{p,q}(\cdot)$) which satisfies the following conditions for all $x, y, z \in X$:

(i) $F_{x,y}(0) = 0$,

(ii) $F_{x,y}(t) = 1$ *for all* $t > 0$ *iff* $x = y$,

(iii) $F_{x,y}(t) = F_{y,x}(t), t > 0$,

(iv) *if* $F_{x,y}(t_1) = 1$ *and* $F_{y,z}(t_2) = 1$, *then* $F_{x,z}(t_1 + t_2) = 1$.

Definition 3. ***t-norm [14, 25]***
A binary operation $T : [0, 1] \times [0, 1] \to [0, 1]$ is called a triangular norm (abbreviated t-norm) if the following conditions are satisfied:

(i) $T(1, a) = a$,

(ii) $T(a, b) = T(b, a)$,

(iii) $T(c,d) \geq T(a,b)$ *whenever* $c \geq a$ *and* $d \geq b$,

(iv) $T(T(a,b),c) = T(a,T(b,c))$.

Definition 4. ***Menger Space [14, 25]***
A Menger space is a triplet (X, F, T) *where* X *is a non empty set,* F *is a function defined on* $X \times X$ *to the set of distribution functions and* T *is a t-norm, such that the following are satisfied:*

(i) $F_{x,y}(0) = 0$ *for all* $x, y \in X$,

(ii) $F_{x,y}(s) = 1$ *for all* $s > 0$ *and* $x, y \in X$ *if and only if* $x = y$,

(iii) $F_{x,y}(s) = F_{y,x}(s)$ *for all* $x, y \in X, s > 0$ *and*

(iv) $F_{x,y}(u+v) \geq T(F_{x,z}(u), F_{z,y}(v))$ *for all* $u, v \geq 0$ *and* $x, y, z \in X$.

Definition 5. ***Fuzzy Metric Space (Kramosil and Michalek) [18]***
The 3-tuple (X, M, T) *is said to be a fuzzy metric space if* X *is an arbitrary set,* T *is a t-norm and* M *is a fuzzy set on* $X^2 \times [0, \infty)$ *satisfying the following conditions*

(i) $M(x,y,0) = 0$,

(ii) $M(x,y,t) = 1$ *for all* $t > 0$ *if and only if* $x = y$,

(iii) $M(x,y,t) = M(y,x,t)$,

(iv) $M(x,z,t+s) \geq T(M(x,y,t), M(y,z,s))$,

(v) $M(x,y,.) : [0,\infty) \to [0,1]$ *is left continuous for* $x, y, z \in X$ *and* $t, s > 0$.

George and Veeramani have extended fuzzy metric space in order to ensure a Hausdorff topology on the fuzzy metric space, in [11]. The definition is as follows :

Definition 6. ***Fuzzy Metric Space (George and Veeramani) [11]***
The 3-tuple (X, M, T) *is said to be a fuzzy metric space if* X *is an arbitrary set,* T *is a continuous t-norm and* M *is a fuzzy set on* $X^2 \times (0, \infty)$ *satisfying the following conditions:*

(i) $M(x,y,t) > 0$,

(ii) $M(x,y,t) = 1$ *if and only if* $x = y$,

(iii) $M(x,y,t) = M(y,x,t)$

(iv) $M(x,z,t+s) \geq T(M(x,y,t), M(y,z,s))$,

(v) $M(x,y,.) : (0,\infty) \to [0,1]$ *is continuous for* $x, y, z \in X$ *and* $t, s > 0$.

Definition 7. ***Convergent Sequence[25]***
A sequence $\{x_n\}$ in a fuzzy metric space (X, M, T) is said to be convergent to $x \in X$ if $\lim_{n\to\infty} M(x_n, x, t) = 1$, for each $t > 0$.

Definition 8. ***Cauchy Sequence[11]***
A sequence $\{x_n\}$ in a fuzzy metric space (X, M, T) is called Cauchy sequence if for $\lambda \in (0, 1)$ and $t > 0$ there exists a positive integer N_1 such that $M(x_m, x_n, t) > 1-\lambda$ for all $m, n \geq N_1$.

Definition 9. ***G-Cauchy Sequence[12]***
A sequence $\{x_n\}$ in a fuzzy metric space (X, M, T) is called G-Cauchy sequence if for $\lambda \in (0, 1)$ and $t > 0$ there exists a positive integer N_1 such that $M(x_{n+p}, x_n, t) > 1 - \lambda$ for all $n \geq N_1$ and $p > 0$.

It follows immediately that a Cauchy sequence is a G-Cauchy sequence but the converse is not always true. This has been established by an example in [28].

Definition 10. *A fuzzy metric space (X, M, T) is said to be complete if every Cauchy sequence in X converges in X.*

Definition 11. *[26] Let (X, F) is a probabilistic metric space and f is a self-mapping on X. The mapping f is said to be a B-contraction (or Sehgal contraction) if*

$$F_{fp,fq}(kt) \geq F_{p,q}(t) \ \forall p, q \in X \text{ and } \forall t > 0,$$

where $0 < k < 1$ is a fixed constant.

As already mentioned in the introduction another notion of contraction known as C-contraction in probabilistic metric spaces was introduced by Hicks [16]. C-contractions in probabilistic and fuzzy metric spaces have been considered in a number of works such as those noted in [15, 20, 21] and [29].

In 1984 Khan et al [17] introduced the following notion, which they called alternating distance function and using it they had proved some fixed point theorems in a complete metric spaces.

Definition 12. ***Altering distance function [17]***
An altering distance function is a function $\psi : [0, \infty) \to [0, \infty)$
(i) which is monotone increasing and continuous and
(ii) $\psi(t) = 0$ if and only if $t = 0$.

They proved the following result.

Theorem 13. *[17] Let (X, d) be a complete metric space, ψ be an altering distance function and let $f : X \to X$ be a self mapping which satisfies the following inequality*

$$\psi(d(fx, fy)) \leq c\psi(d(x, y))$$

for all $x, y \in X$ and for some $0 < c < 1$. Then f has a unique fixed point.

To extending the above idea in the context of Menger spaces Choudhury and Das [5] introduced the following definition.

Definition 14. ***Φ-function*** *[5]*
A function $\phi : R \to R^+$ is said to be a Φ-function if it satisfies the following conditions:

(i) $\phi(t) = 0$ *if and only if* $t = 0$,
(ii) $\phi(t)$ *is increasing and* $\phi(t) \to \infty$ *as* $t \to \infty$,
(iii) ϕ *is left continuous in* $(0, \infty)$,
(iv) ϕ *is continuous at* 0.

Definition 15. *Let (X, M, T) be a fuzzy metric space. A self map $f : X \to X$ is said to be ϕ-contractive if*

$$M(fx, fy, \phi(t)) \geq M(x, y, (\phi(\frac{t}{c})), \tag{2.1}$$

where $0 < c < 1$, $x, y \in X$, $t > 0$ and ϕ is a Φ-function.

Definition 16. *Let (X, M, T) be a fuzzy metric spaces. A mapping $f : X \to X$ is called a generalized C-contraction if for any $\epsilon > 0$ and $\lambda > 0$,*

$$M(x, y, \phi(\epsilon)) > 1 - \lambda \text{ implies } M(fx, fy, \phi(k\epsilon)) > 1 - k_1\lambda, \tag{2.2}$$

where ϕ is a Φ-function and k, k_1 are positive numbers with $0 < k, k_1 < 1$.

Definition 17. *Let (X, M, T) be a fuzzy metric space. A sequence $\{x_n\}$ in X is said to be point convergent or p-convergent to $x \in X$ if there exists $t > 0$ such that*

$$\lim_{n\to\infty} M(x_n, x, t) = 1.$$

We write $x_n \to_p x$ and call x as the p-limit of $\{x_n\}$.

It follows that convergence implies p-convergence. That the converse is not true has been established by an example in [21].

The following lemma was proved in [21].

Lemma 18. *p-limit of a point convergent sequence is unique.*

3 Main Results

Theorem 19. *Let (X, M, T) be a fuzzy metric space in the sense of George and Veeramani and $f : X \to X$ be a ϕ-contraction. Suppose that for some $x_0 \in X$ the sequence $\{f^n x_0\}$ has a p-convergent subsequence. Then f has a unique fixed point.*

Proof. Let $x_0 \in X$. In view of the condition (i) and (iv) in definition 2.14, for $s > 0$ we can find a number r such that $s > \phi(r)$. Then for $s > 0$ we have by (1),

$$\begin{aligned} M(x_n, x_{n+1}, s) &\geq M(fx_{n-1}, fx_n, \phi(r)) \\ &\geq M(x_{n-1}, x_n, \phi(\frac{r}{c})) \\ &= M(fx_{n-2}, fx_{n-1}, \phi(\frac{r}{c})) \\ &\geq M(x_{n-2}, x_{n-1}, \phi(\frac{r}{c^2})) \\ &\geq \cdots \geq M(x_0, x_1, \phi(\frac{r}{c^n})). \end{aligned}$$

Therefore for all $n \geq 1$,

$$M(x_n, x_{n+1}, s) \geq M(x_0, x_1, \phi(\frac{r}{c^n})).$$

Taking $n \to \infty$, we have for all $s > 0$,

$$M(x_n, x_{n+1}, s)) \to 1. \tag{3.1}$$

Suppose $\{x_{n_j}\}$ is a p-convergent subsequence of $\{x_n\}$, therefore there is a $y_0 \in X$ and $\epsilon > 0$ such that $M(x_{n_j}, y_0, \frac{\epsilon}{2}) \to 1$. Hence for $\lambda \in (0, 1)$, we can find a positive integer $N_1(\lambda)$ such that

$$M(x_{n_j}, y_0, \frac{\epsilon}{2}) > 1 - \lambda, \quad \forall j \geq N_1(\lambda). \tag{3.2}$$

Now we show that $M(x_{n_j+1}, y_0, \epsilon) \to 1$. Since T is continuous, there is a $\delta \in (0, 1)$ such that $T(1 - \delta, 1 - \delta) > 1 - \lambda$. By virtue of (3) we can find a positive integer N_2 depend on δ and hence depend on λ such that,

$$M(x_{n_j}, x_{n_j+1}, \frac{\epsilon}{2}) > 1 - \delta \quad \forall j \geq N_2(\lambda). \tag{3.3}$$

Then for all $j > max\{N_1(\lambda), N_2(\lambda)\}$ we have,

$$\begin{aligned} M(x_{n_j+1}, y_0, \epsilon) &\geq T(M(x_{n_j+1}, x_{n_j}, \frac{\epsilon}{2}), M(x_{n_j}, y_0, \frac{\epsilon}{2})) \\ &\geq T(1 - \delta, 1 - \delta) \quad (by\ (4)\ and\ (5)) \\ &> 1 - \lambda. \end{aligned}$$

Hence, $\lim_{j\to\infty} M(x_{n_j+1}, y_0, \epsilon) = 1$, that is,

$$x_{n_j+1} \xrightarrow{p} y_0. \tag{3.4}$$

We now show that $M(x_{n_j+1}, fy_0, \epsilon) = 1$, as $n \to \infty$. By the property of ϕ-function we can find $\epsilon_1 > 0$ such that $\frac{\epsilon}{2} < \phi(\epsilon_1)$.

Now, we have,

$$\begin{aligned} M(x_{n_j+1}, fy_0, \phi(\epsilon_1)) &= M(fx_{n_j}, fy_0, \phi(\epsilon_1)) \\ &\geq M(x_{n_j}, y_0, \phi(\frac{\epsilon_1}{c})) \quad (by\ \ (1)) \\ &\geq M(x_{n_j}, y_0, \phi(\epsilon_1)) \\ &\geq M(x_{n_j}, y_0, \frac{\epsilon}{2}) \\ &\to 1 \quad as \quad j \ \to \ \infty. \end{aligned}$$

Therefore, $x_{n_j+1} \xrightarrow{p} fy_0$. Again by Lemma 2.18 we have p-limit of a p-convergent sequence is unique. Therefore, we have $fy_0 = y_0$, that is, y_0 is a fixed point of f.

We next show that the fixed point is unique. If possible, let u and v be two fixed points of f. As in the above corresponding to a given $s_1 > 0$, we can find a $r_1 > 0$ such that $s_1 > \phi(r_1)$. Then we have,

$$\begin{aligned} M(u, v, s_1) &= M(fu, fv, s_1) \\ &\geq M(fu, fv, \phi(r_1)) \\ &\geq M(u, v, \phi(\frac{r_1}{c})) \\ &= M(fu, fv, \phi(\frac{r_1}{c})) \\ &\geq M(u, v, \phi(\frac{r_1}{c^2})) \\ &\geq \dots \ \dots \\ &\geq M(u, v, \phi(\frac{r_1}{c^n})). \end{aligned}$$

Taking $n \to \infty$ we have $M(u, v, s_1) \to 1$ for all $s_1 > 0$, that is, $u = v$. This proves the uniqueness of the fixed point and completes the proof.

Theorem 20. *Let (X, M, T) be a fuzzy metric space in the sense of George and Veermani and $f : X \to X$ be a generalized C-contraction. Suppose that for some $x \in X$ the sequence $\{f^n x\}$ has a p-convergent subsequence. Then f has a unique fixed point.*

Proof Let f satisfy (2) and $t > 1$. Now for any $r > 0$,

$$M(x, y, \phi(r)) > 1 - t \quad for \;\; all \;\; x, y \in X.$$

Then we have,

$$M(fx, fy, \phi(kr)) > 1 - k_1 t.$$

Applying the above procedure we have after n steps,

$$M(f^n x, f^n y, \phi(k^n r)) > 1 - k_1^n t. \tag{3.5}$$

Let $\epsilon > 0, \lambda > 0$ be arbitrary. Since $0 < k, k_1 < 1$, we have $k_1^n t \to 0$ as $n \to \infty$, therefore there exists a positive integer $N_1(\lambda)$ such that for all $n > N_1(\lambda)$

$$1 - k_1^n t > 1 - \lambda. \tag{3.6}$$

Again by the properties of ϕ-function we can find a positive integer $N_2(\epsilon)$ such that

$$\epsilon > \phi(k^n r), \quad \forall \; n > N_2(\epsilon). \tag{3.7}$$

Using (8) and (9) we have from (7), for all $x, y \in X$

$$M(f^n x, f^n y, \epsilon) \geq M(f^n x, f^n y, \phi(k^n r)) > 1 - k_1^n t > 1 - \lambda. \tag{3.8}$$

Therefore, for all $n > N(\epsilon, \lambda) = \max\{N_1(\lambda), N_2(\epsilon)\}$, we have

$$M(f^n x, f^n y, \epsilon) > 1 - \lambda. \tag{3.9}$$

Putting $x = x_0$ and $y = x_{m-n}$ for $m \geq n$ in (11) we have that $M(x_n, x_m, \epsilon) > 1 - \lambda$. Hence, $\{f^n x\}$ is a Cauchy sequence.

Suppose $\{x_n\}$ has a p-convergent subsequence $\{x_{n_j}\}$ which converges to some point $y_0 \in X$. Then there exists $\lambda > 0$ and a positive integer $N_3(\lambda)$ such that for all $j > N_3(\lambda)$, $M(x_{n_j}, y_0, \frac{\epsilon}{2}) > 1 - \lambda$.

Since $\{x_n\}$ and then $\{x_{n_j}\}$ are Cauchy sequences we can take $N_0 = \max\{N_1(\lambda), N_2(\epsilon), N_3(\lambda)\}$ such that

$$M(x_{n_j+1}, x_{n_j}, \frac{\epsilon}{2}) > 1 - \lambda, \;\; \forall j \geq N_0 \tag{3.10}$$

and from the p-convergent subsequence we have,

$$M(x_{n_j}, y_0, \frac{\epsilon}{2}) > 1 - \lambda, \;\; \forall j \geq N_0. \tag{3.11}$$

Since T is continuous, we can find a $\delta \in (0,1)$ such that $T(1-\lambda, 1-\lambda) > 1-\delta$. Now we have,

$$\begin{aligned} M(x_{n_j+1}, y_0, \epsilon) &\geq T(M(x_{n_j+1}, x_{n_j}, \frac{\epsilon}{2}), M(x_{n_j}, y_0, \frac{\epsilon}{2})) \\ &\geq T(1-\lambda, 1-\lambda) \quad (by \ (12) \ and \ (13)) \\ &> 1-\delta, \end{aligned}$$

which implies that $x_{n_j+1} \xrightarrow{p} y_0$.

By the properties of ϕ we can get $\epsilon_1 > 0$ such that $\phi(\epsilon_1) \geq \frac{\epsilon}{2} \geq \phi(k\epsilon_1)$ and we have by (13), for all $j > N_0$

$$M(x_{n_j}, y_0, \phi(\epsilon_1)) \geq M(x_{n_j}, y_0, \tfrac{\epsilon}{2}) > 1-\lambda.$$

Therefore by (2) we have,

$$M(fx_{n_j}, fy_0, \phi(k\epsilon_1)) > 1 - k_1\lambda, \text{ for all } j > N_0$$

that is, $M(x_{n_j+1}, fy_0, \phi(k\epsilon_1)) > 1-\lambda$, (since $0 < k_1 < 1$),
that is, $M(x_{n_j+1}, fy_0, \frac{\epsilon}{2}) \geq M(x_{n_j+1}, fy_0, \phi(k\epsilon_1)) > 1-\lambda$, for all $j > N_0$.
Therefore, $x_{n_j+1} \xrightarrow{p} fy_0$. Again by Lemma 2.18 we have p-limit of a p-convergent sequence is unique. Therefore, we have $fy_0 = y_0$, that is, y_0 is a fixed point of f.

For uniqueness, let u and v be two fixed points of f, then by the properties of ϕ-function we can find r_1 and t_1 with $r_1 > 0$ and $0 < t_1 < 1$ such that

$$M(u, v, \phi(r_1)) > 1 - t_1.$$

Therefore by (2) we have

$$M(fu, fv, \phi(kr_1)) > 1 - k_1 t_1$$

that is $M(u, v, \phi(kr_1)) > 1 - k_1 t_1$. Applying this procedure we have after n steps

$$M(u, v, \phi(k^n r_1)) > 1 - k_1^n t_1.$$

Again let $\epsilon > 0$ be arbitrary. By the properties of ϕ-function we can find a positive integer N_4 such that $\epsilon > \phi(k^n r_1)$ for all $n > N_4$. Therefore $M(u, v, \epsilon) > 1 - k_1^n t_1$ for all $n > N_4$. Therefore $M(u, v, \epsilon) = 1$ for arbitrary $\epsilon > 0$, that is $u = v$.

Example 21. *Let* $X = \{x_1, x_2, x_3\}$, $M(x, y, t)$ *be defined as*

$$M(x_1, x_2, t) = M(x_2, x_1, t) = \begin{cases} 0, & if \quad t \leq 0, \\ 0.9, & if \quad 0 < t \leq 3, \\ 1, & if \quad t > 3, \end{cases}$$

$$M(x_1, x_3, t) = M(x_3, x_1, t) =$$

$$M(x_2, x_3, t) = M(x_3, x_2, t) = \begin{cases} 0, & if \quad t \leq 0, \\ 0.7, & if \quad 0 < t < 6, \\ 1, & if \quad t \geq 6, \end{cases}$$

and $T(a, b) = min\{a, b\}$ *then* (X, M, T) *is a complete fuzzy metric space. If* $fx_1 = fx_2 = x_2, fx_3 = x_1$ *and* $\phi(t) = \sqrt{t}$ *then* f *satisfies all the conditions of Theorem 3.2 and* x_2 *is the unique fixed point of* f.

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