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n-JORDAN MULTIPLIERS

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Abstract. Let A be a Banach algebra, X be a Banach left A-module and $n \geq 2$ be an integer. A bounded linear operator $T : A \longrightarrow X$ is called an n-Jordan multiplier if for each $a \in A$, $T(a^n) = a \cdot T(a^{n-1})$. In this paper we investigate this notion and give some illuminating examples. Also, we give an approximate local version of n-Jordan multipliers and try to investigate when an approximate local n-Jordan multiplier is an n-Jordan multiplier. Finally, for functional Banach algebras we give a characterization of n-Jordan multipliers.

1 Introduction and preliminaries

The theory of multipliers for the first time introduced and studied by Helgason in [5]. Also, Wang in [10] investigated and studied this notion and proved some remarkable results of multipliers. Indeed, for a Banach algebra A, a linear operator $T : A \longrightarrow A$ is a (right) multiplier if $T(ab) = aT(b)$ for all $a, b \in A$.

On the other hand, Hejazian et al., introduced the concept of n-homomorphisms for integers $n \geq 2$; see [4]. Also, Gordji in [3] introduced the theory of n-Jordan homomorphisms and gave a nice relation between 3-homomorphisms and 3-Jordan homomorphisms.

Using the idea of n-homomorphisms, Laali and the author of the paper in [7], introduced and studied the notion of n-multipliers and gave a nice relation of this notion with n-homomorphisms.

In this paper we introduce and investigate the notion of n-Jordan multiplier from a Banach algebra A into a Banach left A-module X. In the sequel of this section we give some preliminaries which will be used later. For undefined concepts we refer the reader to [2].

Definition 1. *A Banach algebra A is called nilpotent if there exists an integer $n \geq 2$ such that*

$$A^n = \{a_1 a_2 a_3 ... a_n : a_1, a_2, a_3, ..., a_n \in A\} = \{0\}.$$

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The index of A, denoted by $I(A)$, is the minimum $n \in \mathbb{N}$ such that $A^n = \{0\}$. So, if $I(A) = n$, there exists elements $a_1, a_2, ...a_{n-1} \in A$ such that $a_1 a_2 ... a_{n-1} \neq 0$.

Definition 2. *Let A be an Banach algebra. We say that A is nil if there exists $n \in \mathbb{N}$ such that $a^n = 0$ for all $a \in A$. The nil index of A, denoted by $NI(A)$ is the minimum $n \in \mathbb{N}$ such that $a^n = 0$ for all $a \in A$.*

Theorem 3. *(*Grabiner) Let A be a nil (F)-algebra. Then A is nilpotent.

Proof. See [2, Theorem 2.6.34]. □

To see the definition of an (F)-algebra see [2, Definition 2.2.5]. As an example, every Banach algebra is an (F)-algebra.

Let A be a Banach algebra and $a, b \in A$. Define a bounded bilinear functional on $A^* \times A^*$ as

$$(a \otimes b)(f, g) = f(a)g(b) \qquad (f, g \in A^*).$$

The projective tensor product space $A \widehat{\otimes} A$ with the above multiplication, natural addition and the norm

$$\|x\| = \inf \left\{ \sum_{n=1}^{\infty} \|a_n\| \|b_n\| < \infty : x = \sum_{n=1}^{\infty} a_n \otimes b_n \right\},$$

is a Banach algebra that is characterized as follows;

$$\left\{ \sum_{n=1}^{\infty} a_n \otimes b_n : n \in \mathbb{N}, a_n, b_n \in A, \sum_{n=1}^{\infty} \|a_n\| \|b_n\| < \infty \right\}.$$

Clearly, $A \widehat{\otimes} A$ with the following action is a Banach left A-module;

$$a \cdot (b \otimes c) = ab \otimes c \qquad (a, b, c \in A).$$

2 n-Jordan multipliers

Let A be a Banach algebra and X be a Banach left A-module. Recall that a bounded linear map $T : A \to X$ is called a right Jordan multiplier if $T(a^2) = a.T(a)$ for each $a \in A$. In the rest we drop the prefix right for simplicity. We give the following definition of an n-Jordan multiplier as a generalization of Jordan multipliers.

Definition 4. *Let A be a Banach algebra, X be a Banach left A-module and let $n \geq 2$ be an integer. A bounded linear map $T : A \to X$ is an n-Jordan multiplier if*

$$T(a^n) = a \cdot T(a^{n-1}) \qquad (a \in A).$$

Clearly, each Jordan multiplier is an n-Jordan multiplier but the converse is not valid in general (see Example 5 below). We denote by $\mathrm{JMul}_n(A,X)$ the set of all n-Jordan multipliers from A into X and suppose that $\mathrm{JMul}_n(A)=\mathrm{JMul}_n(A,A)$. It is clear that $\mathrm{JMul}_n(A,X)$ is a vector subspace of $B(A,X)$; the Banach space of all bounded linear operators from A into X, and one can see that it is closed, because if $\{T_m\}$ is a sequence in $\mathrm{JMul}_n(A,X)$ for which $T_n\longrightarrow T$ where $T\in B(A,X)$, then for each $a\in A$ we have

$$\begin{aligned}\|T(a^n)-a\cdot T(a^{n-1})\| &\leq \|T(a^n)-T_m(a^n)\|+\|T_m(a^n)-a\cdot T(a^{n-1})\|\\ &\leq \|T-T_m\|\|a^n\|+\|a\cdot T_m(a^{n-1})-a\cdot T(a^{n-1})\|\\ &\leq \|T-T_m\|\|a^n\|+\|T-T_m\|\|a\|\|a^{n-1}\|.\end{aligned}$$

If $m\to\infty$, the right hand side of the above inequalities tend to zero and hence $T(a^n)-a\cdot T(a^{n-1})$. So, T is an n-Jordan multiplier. Hence, $\mathrm{JMul}_n(A,X)$ is closed.

Therefore, $\mathrm{JMul}_n(A,X)$ is a Banach space for every positive integer $n\geq 2$.

Following the notations of [7], let $\mathrm{Mul}_n(A,X)$ show the set of all n-multipliers from A into X. Note that $T\in\mathrm{Mul}_n(A,X)$ if

$$T(a_1a_2...a_n)=a_1\cdot T(a_2...a_n)\quad (a_1,a_2,a_3,...,a_n\in A).$$

The following example shows the difference between 3-Jordan multipliers and Jordan multipliers.

Example 5. *Suppose that A defined as follows*

$$A=\begin{bmatrix}0 & \mathbb{R} & \mathbb{R} & \mathbb{R}\\ 0 & 0 & \mathbb{R} & \mathbb{R}\\ 0 & 0 & 0 & \mathbb{R}\\ 0 & 0 & 0 & 0\end{bmatrix}.$$

Clearly, A is a Banach algebra equipped with the usual matrix-like operations and l_1-norm, that is, the sum of all absolute values of entries. Define the operator $T:A\longrightarrow A$ as follows

$$T\left(\begin{bmatrix}0 & a & b & c\\ 0 & 0 & d & e\\ 0 & 0 & 0 & f\\ 0 & 0 & 0 & 0\end{bmatrix}\right)=\begin{bmatrix}0 & f & b & c\\ 0 & 0 & d & e\\ 0 & 0 & 0 & a\\ 0 & 0 & 0 & 0\end{bmatrix}$$

where $\mathfrak{a}=\begin{bmatrix}0 & a & b & c\\ 0 & 0 & d & e\\ 0 & 0 & 0 & f\\ 0 & 0 & 0 & 0\end{bmatrix}$ *is an arbitrary element of A. Clearly, T is a bounded* *linear operator on A,* $\mathfrak{a}^2 = \begin{bmatrix} 0 & 0 & ad & ae+bf \\ 0 & 0 & d & df \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ *and* $\mathfrak{a}^3 = \begin{bmatrix} 0 & 0 & 0 & adf \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

Therefore, $T(\mathfrak{a}^2) = \mathfrak{a}^2$ *and* $T(\mathfrak{a}^3) = \mathfrak{a}^3$. *Now, immediately one can see that T is a 3-Jordan multiplier but it is not a Jordan multiplier.*

In the following proposition we show that the class of n-Jordan multipliers is strictly larger than the class of n-multipliers.

Proposition 6. *There exists a Banach algebra A and a Banach left A-module X such that*

$$\mathrm{Mul}_n(A,X) \subsetneq \mathrm{JMul}_n(A,X) \qquad (n = 3,4,5,...).$$

Moreover, there exists a Banach algebra A and a Banach left A-module X such that for every positive integer $n \geq 3$,

$$\mathrm{JMul}_{n-1}(A,X) \subsetneq \mathrm{JMul}_n(A,X).$$

Proof. For each Banach algebra A and Banach left A-module X, clearly

$$\mathrm{Mul}_n(A,X) \subseteq \mathrm{JMul}_n(A,X).$$

Now, let A be a nil Banach algebra such that $NI(A) = n$. So, A is nilpotent by Grabiner's Theorem. Suppose that $I(A) = m$ and $m > n$. Therefore, there exists $a_1, a_2, ...a_{m-1}$ in A such that $a_1 a_2 ... a_{m-1} \neq 0$. Take $X = A\widehat{\otimes}A$ and let $T : A \to X$ be an operator specified by

$$T(a) = a_1 a_2 ... a_{m-1} \otimes a \qquad (a \in A). \tag{2.1}$$

The operator T is an element of $\mathrm{JMul}_n(A,X)$ which is not belong to $\mathrm{Mul}_n(A,X)$. Because,

$$T(a^n) = a_1 a_2 ... a_{m-1} \otimes a^n = 0,$$
$$a.T(a^{n-1}) = a(a_1 a_2 ... a_{m-1} \otimes a^{n-1}) = a a_1 a_2 ... a_{m-1} \otimes a^{n-1} = 0.$$

and $T(a_1 a_2 ... a_n) = a_1 a_2 ... a_{m-1} \otimes a_1 a_2 ... a_n \neq 0 = a_1 \cdot T(a_2 a_3 ... a_n)$.

For the second part of the theorem, take the Banach algebra A and X as above and let T be the operator defined by equation 2.1. It is clear that every $(n-1)$-Jordan multiplier is an n-Jordan multiplier. On the other hand, there exists an element $a_0 \in A$ such that $a_0^{n-1} \neq 0$. Hence, $T(a_0^{n-1}) = a_1 a_2 ... a_{m-1} \otimes a_0^{n-1} \neq 0 = a_0 \cdot T(a_0^{n-2})$ and this show the strict inclusion. □

Let $\{A_n : n \in \mathbb{N}\}$ be a collection of algebras. Suppose that $\prod_n A_n$ denotes the product space of the collection $\{A_n : n \in \mathbb{N}\}$ such that the linear operations being given coordinatewise. We recall that the c_0-direct sum of the collection is

$$\bigoplus_n^0 A_n = \{(a_n) \in \prod_n A_n : \lim_n a_n = 0, \|(a_n)\|_\infty = \sup \|a_n\|_{A_n} < \infty\}.$$

Remark 7. *There exists a Banach algebra A and a Banach left A-module X such that A is not nil and $\mathrm{JMul}_2(A,X) \subsetneq \mathrm{JMul}_3(A,X)$. To see this, let $\{A_n\}$ be a collection of Banach algebras such that A_1 is nil with $NI(A_1) = 3$. So, by Grabiner's Theorem, there exists $m \in \mathbb{N}$ such that $I(A_1) = m$. Also, let $A = \bigoplus_n^0 A_n$ and $X = A\widehat{\otimes}A$. There exists $c_1, \dots c_{m-1} \in A_1$ with $c_1c_2 \dots c_{m-1} \neq 0$. For $1 \le i \le m-1$, put $a_i = (c_i, 0, \dots)$ and $\mathfrak{a} = a_1a_2 \dots a_{m-1}$. Define the operator $T : A \longrightarrow X$ by*

$$T((b_n)) = \mathfrak{a} \otimes (b_n) \qquad ((b_n) \in A).$$

Now, it is easy to check that $T \in \mathrm{JMul}_3(A,X) \setminus \mathrm{JMul}_2(A,X)$ and A is not a nil Banach algebra in general.

3 Approximate local n-Jordan multipliers

Suppose that $T : A \to X$ is a bounded linear operator such that X is a Banach left A-module. We say that the operator T is an approximate local n-Jordan multiplier if, for each $a \in A$, there exist a sequence $\{T_{a,m}\}$ in $\mathrm{JMul}_n(A,X)$ such that, $T(a) = \lim_m T_{a,m}(a)$. Samei in [9], investigated approximate local multipliers and answered this question; When an approximate local multiplier is a multiplier? In this section we answer this question in the setting of n-Jordan multipliers.

To proceed further first we recall the algebraic reflexivity from [1]. Let X and Y be Banach spaces and S be a subset of $B(X,Y)$. Put

$$\mathrm{ref}(S) = \{T \in B(X,Y) : T(x) \in \overline{\{s(x) : s \in S\}} \ \ \forall x \in X\}.$$

Then S is algebraically reflexive if, $S = \mathrm{ref}(S)$ or just $\mathrm{ref}(S) \subseteq S$.

Lemma 8. *Let A be a Banach algebra, X be a Banach left A-module and $n \ge 3$, be an integer. Then the following statements are equivalent.*

1. *Every approximate local n-Jordan multiplier from A into X is an n-Jordan multiplier.*
2. JMul_n*(A,X) is algebraically reflexive.*

Proof. (1) $\Rightarrow$ (2): Let $T \in \mathrm{ref}(\mathrm{JMul}_n(A,X))$. So, for each $a \in A$ there exists a sequence $\{T_{a,m}\}$ in $\mathrm{JMul}_n(A,X)$ such that, $T(a) = \lim_m T_{a,m}(a)$. Hence, T is an approximate local n-Jordan multiplier. Therefore, T is an n-Jordan multiplier by assumption and this shows that JMul_n(A,X) is algebraically reflexive.

(2) $\Rightarrow$ (1): Let $T : A \to X$ be an approximate local n-Jordan multiplier. So, for each $a \in A$, there exists a sequence $\{T_{a,m}\}$ such that, $T(a) = \lim_m T_{a,m}(a)$. Hence, $T \in \mathrm{ref}(\mathrm{JMul}_n(A, X))$ and reflexivity of $\mathrm{JMul}_n(A, X)$ implies that T is an n-Jordan multiplier. $\square$

Let A be a Banach algebra and X be a Banach left A-module. Then for each $x \in X$, the left annihilator of x in A is defined by $x^\perp = \{a \in A : a \cdot x = 0\}$.

Theorem 9. *Suppose that A is a Banach algebra such that $\mathrm{JMul}_n(A, A^*)$ is algebraically reflexive and X is a Banach left A-module such that $\{x \in X : x^\perp = A\} = 0$. Then every approximate local n-Jordan multiplier from A into X is an n-Jordan multiplier.*

Proof. Let $T : A \to X$ be an approximate local n-Jordan multiplier and $f \in X^*$. Define a map $\mathfrak{K}_f : X \to A^*$ as follows

$$\mathfrak{K}_f(x) = x \bullet f \qquad (x \in X),$$

where $x \bullet f \in A^*$ is defined by $x \bullet f(a) = f(a \cdot x)$ for all $a \in A$. Therefore, $\mathfrak{K}_f$ is a bounded left A-module morphism. Because, for $a \in A$ and $x \in X$ we have

$$\mathfrak{K}_f(a \cdot x) = (a \cdot x) \bullet f = a \cdot (x \bullet f) = a \cdot \mathfrak{K}_f(x).$$

So, using Lemma 8, we conclude that $\mathfrak{K}_f \circ T \in \mathrm{JMul}_n(A, A^*)$.

Now, for $a \in A$ we have

$$\begin{aligned} \mathfrak{K}_f(T(a^n)) = \mathfrak{K}_f \circ T(a^n) &= a \cdot \mathfrak{K}_f \circ T(a^{n-1}) \\ &= a \cdot \mathfrak{K}_f(T(a^{n-1})) \\ &= \mathfrak{K}_f(a \cdot T(a^{n-1})). \end{aligned}$$

Therefore, $\mathfrak{K}_f(T(a^n) - a \cdot T(a^{n-1})) = 0$. If we put $u = T(a^n) - a \cdot T(a^{n-1})$, then $f(a \cdot u) = 0$ for all $a \in A$. So, by the Hahn-Banach theorem we have $a \cdot u = 0$ for all $a \in A$. So, $u^\perp = A$ and this implies that $u = 0$. Hence, T is an n-Jordan multiplier. $\square$

4 Characterization on functional Banach algebra

Let $(A, \|\cdot\|)$ be a non-empty Banach space and $0 \neq f \in A^*$. For each $a, b \in A$ define, $a \circ b = f(b)a$. One can easily check that A with the multiplication "$\circ$" and the norm $\|\cdot\|$ is a Banach algebra called the functional Banach algebra which will be denoted by A_f; see [8] and [6] for more details. For each $a \in A$, let $a^n = \overbrace{a \circ a \circ a \circ \ldots \circ a}^{n\ times}$.

Theorem 10. *Let f be an injective functional,* $\dim(A) > 1$ *and* $T : A_f \to A_f$ *be a bounded linear operator. Then the following assertions are equivalent.*

1. *T is an n-Jordan multiplier.*
2. *$T(a) \circ a = a \circ T(a)$ for all $a \in A$.*
3. *$T(a) \circ b = b \circ T(a)$ for all $a, b \in A$.*

Proof. (1) $\Rightarrow$ (2): Let $a \in A$ and $T(a) \circ a = a \circ T(a)$. So, we have

$$\begin{aligned} T(a^n) = T(f(a)^{n-1}a) = f(a)^{n-1}T(a) &= f(a)^{n-2}(T(a) \circ a) \\ &= f(a)^{n-2}(a \circ T(a)) = a \circ (f(a)^{n-2}T(a)) \\ &= a \circ T(a^{n-1}). \end{aligned}$$

Therefore, T is an n-Jordan multiplier.

(2) $\Rightarrow$ (1): Let T be an n-Jordan multiplier. So, we have

$$\begin{aligned} f(a)^{n-2}(T(a) \circ a) = f(a)^{n-1}T(a) = T(a^n) &= a \circ T(a^{n-1}) \\ &= af(a)^{n-2}f(T(a)) \\ &= f(a)^{n-2}(a \circ T(a)). \end{aligned}$$

Now, we have two cases; If $f(a) \neq 0$, then $T(a) \circ a = a \circ T(a)$ and if $f(a) = 0$, the injectivity of f yields $a = 0$. Therefore $T(a) = 0$ and hence $f(T(a)) = 0$. So $T(a) \circ a = a \circ T(a)$, which completes the proof.

(3) $\Rightarrow$ (2): This is clear.

(2) $\Rightarrow$ (3): The Banach algebra A_f is a semiprime ring, i.e., if $a \in A$ and $aA_fa = \{0\}$, then $a = 0$. Since, $\dim(A) > 1$ we conclude that the characteristic of A_f is not two, i.e., the minimum number such that $a^n = e$ is not two (e is the identity element of A_f). Since $T(a) \circ a = a \circ T(a)$ for all $a \in A$, we conclude that T is a Jordan multiplier. Therefore, by [11, Proposition 1.4], T is a multiplier. Now, with the same argument as in the above for each $a, b \in A$ we have $T(a) \circ b = b \circ T(a)$. □

Example 11. *Let* $\dim(A) > 1$ *and f be injective. For a fixed $a_0 \in A$, define $T : A_f \longrightarrow A_f$ by $T(a) = a_0 \circ a$. Clearly, T is a bounded linear functional. If for each $0 \neq a \in A$, $a \circ T(a) = T(a) \circ a$, then $a = \frac{f(a)}{f(a_0)}a_0$. Hence* $\dim(A) = 1$ *which contradicts the hypothesis. Therefore, T is not an n-Jordan multiplier by Theorem 10.*

Remark 12. *Using the proof of Theorem 10, one can see that*

$$\mathrm{JMul}_2(A_f) = \mathrm{JMul}_n(A_f) = \mathrm{Mul}_n(A_f) = \mathrm{Mul}_2(A_f),$$

for all $n \geq 3$.

Remark 13. *Suspose that A is a non-empty Banach space with the norm $\|\cdot\|$ and $0 \neq f \in A^*$. If we define $a \diamond b = f(a)b$, then A with the the multiplication "$\diamond$" is a Banach algebra which we denote it by ${}_fA$. One can easily check that each bounded linear operator T on ${}_fA$ is an n-Jordan multiplier.*

Two questions

Let A and B be two Banach algebras. A linear map $\varphi : A \to B$ is called an n-Jordan homomorphism if $\varphi(a^n) = \varphi(a)^n$ for all $a \in A$; see [3] for more details.

In [12] Zelazko, proved the following theorem.

Theorem: Let A be a Banach algebra. Also let B be a semisimple commutative Banach algebra. Then each 2-Jordan homomorphism from A into B is a 2-homomorphism.

Gordji in [3] generalized the above theorem for 3-Jordan homomorphism, i.e., he proved that each 3-Jordan homomorphism from a Banach algebra into a semisimple and commutative Banach algebra is a 3-homomorphism.

Now, like the theory of n-Jordan homomorphism we raise the following interesting question for n-Jordan multipliers.

Question 1: Let A be a Banach algebra and X be a Banach left A-module. Let T be an n-Jordan multiplier ($n \geq 3$) from A into X. When T is an n-multiplier? what condition(s) is (are) needed?

Zalar in [11, Corollary 1.5] showed that a linear map on a semisimple algebra A such that $T(a^2) = aT(a)$ for all $a \in A$, is continuous. Now, we raise the following question.

Question 2: Suppose that A is a semisimple Banach algebra, n is an integer with $n \geq 3$ and $T : A \longrightarrow A$ is a linear map such that $T(a^n) = aT(a^{n-1})$. Is T continuous (or equivalently bounded)? what condition(s) is (are) needed?

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