

Summability and Convergence Results of 2-adic Fourier Series

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(Received December 20, 2013; Revised March 15, 2014; Accepted May 27, 2014)

After the presentation of the notions and concepts of 2-adic Fourier analysis we sum up Cesàro summability results of one- and more-dimensional integrable functions. We place this presentation into the context of the same investigations with respect to the trigonometric and the Walsh-Paley systems. Some generalizations (like UDMD systems and Vilenkin-like systems) and other means (like Nörlund logarithmic mean and Marcinkiewicz mean) are also considered. Finally, a transformation method is presented to generate further convergence and summability results while the character system is transformed by a measure-preserving variable-transformation.

Keywords: 2-adic theory, Cesàro means, Fourier series.

AMS Subject Classification: 11E95, 11F85, 43A50, 43A70.

Dedicated to the memory of Professor Dr. L.V.Zhizhiasvili.

1. Notions and concepts

We follow the notions of Schipp-Simon-Wade[22]. Let \mathbb{N} denote the set of natural numbers, $\mathbb{P} := \mathbb{N} \setminus \{0\}$, and let $\mathbb{I} := [0, 1)$ denote the unit interval. Consider the Cartesian products $\mathbb{N}^2 := \mathbb{N} \times \mathbb{N}$ and $\mathbb{I}^2 := \mathbb{I} \times \mathbb{I}$. Denote the 1- and 2-dimensional Haar measure of subsets $E \subseteq \mathbb{I}$ and $F \subseteq \mathbb{I}^2$ by $\mu(E) = |E|$ and $\mu_2(F) = |F|$. Denote the L^p - norm of any function $f \in L^p(\mathbb{I})$ or $f \in L^p(\mathbb{I}^2)$ by $\|f\|_p$. Set $\mathcal{I} := \{[p/2^n, (p+1)/2^n) \mid p, n \in \mathbb{N}, 0 \leq p < 2^n\}$ the set of dyadic intervals. Given $n \in \mathbb{N}$ and $x \in [0, 1)$ let $I_n(x)$ denote the dyadic interval of length 2^{-n} which contains x . Also use the notation $I_n := I_n(0)$ ($n \in \mathbb{N}$). The dyadic expansion of $x \in \mathbb{I}$ is

$$x = \sum_{n=0}^{\infty} x_n 2^{-(n+1)},$$

where $x_n \in \{0, 1\}$. If x is a dyadic rational, that is $x \in \{\frac{p}{2^n} : p, n \in \mathbb{N}, 0 \leq p < 2^n\}$, we choose the expansion which terminates in 0's. Let us mention, that $I_n(x) = \{y \in \mathbb{I} : y_k = x_k \text{ for } k < n\}$. For $x \neq 0$ let $\pi(x) := \min\{n \in \mathbb{N} : x_n = 1\}$, furthermore set $\pi(0) := +\infty$.

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The **2-adic sum** of $a, b \in \mathbb{I}$ is

$$a + b := \sum_{n=0}^{\infty} s_n 2^{-(n+1)},$$

where bits $q_n, s_n \in \{0, 1\}$ ($n \in \mathbb{N}$) are defined recursively as follows:

$$q_{-1} := 0, \quad a_n + b_n + q_{n-1} = 2q_n + s_n \text{ for } n \in \mathbb{N}.$$

The group $(\mathbb{I}, +)$ is a topological group, and it is called the group of 2-adic integers. Furthermore $\|x + y\| \leq \max\{\|x\|, \|y\|\}$, thus a non-Archimedean inequality holds with respect to the norm: $\|x\| := 2^{-\pi(x)}$ for $x \in (0, 1)$, and $\|0\| := 0$. Set

$$v_{2^n}(x) := \exp\left(2\pi i \left(\frac{x_n}{2} + \cdots + \frac{x_0}{2^{n+1}}\right)\right) \quad (x \in \mathbb{I}, n \in \mathbb{N}),$$

and

$$v_n := \prod_{i=0}^{\infty} v_{2^i}^{n_i}, \quad \text{where } n \in \mathbb{N}, \quad n = \sum_{i=0}^{\infty} n_i 2^i \quad (n_i \in \{0, 1\} (i \in \mathbb{N})).$$

It is known that $(v_n, n \in \mathbb{N})$ is the character system of $(\mathbb{I}, +)$.

Denote by

$$\hat{f}(n) := \int_{\mathbb{I}} f \bar{v}_n d\mu, \quad D_n := \sum_{k=0}^{n-1} v_k, \quad S_n := \sum_{k=0}^{n-1} \hat{f}(k) v_k$$

the Fourier coefficients, the Dirichlet kernels and the n -th partial sums of the Fourier series. It is known, that $D_n(x) = v_n(x) \sum_{k=0}^{\infty} n_k (-1)^{x_k} D_{2^k}(x)$ ($n \in \mathbb{P}, k \in \mathbb{N}, x \in I$), where

$$D_{2^k}(x) = \begin{cases} 2^k, & \text{if } x \in I_k \\ 0, & \text{if } x \notin I_k. \end{cases}$$

Let us define the *Cesàro* (or $(C, 1)$) means of Sf by $\sigma_0 f := 0$ and $\sigma_n f := \frac{1}{n} \sum_{k=1}^n S_k f$ ($n \in \mathbb{P}$). Denote by K_n the $(C, 1)$ kernel for $n \in \mathbb{N}$: $K_n := \frac{1}{n} \sum_{i=0}^n D_i$. Now, $\sigma_n f = f * K_n$. Denote by K_n^α the (C, α) kernel for $\alpha \in \mathbb{R}$ and $n \in \mathbb{N}$:

$$K_n^\alpha := \frac{1}{A_n^\alpha} \sum_{i=0}^n A_{n-i}^{\alpha-1} D_i, \quad A_k^\alpha = \frac{(\alpha+1)(\alpha+2) \cdots (\alpha+k)}{k!} \quad (\alpha \neq -k).$$

The (C, α) Cesàro means of the integrable function f is

$$\sigma_n^\alpha f(y) := \frac{1}{A_n^\alpha} \sum_{i=0}^n A_{n-i}^{\alpha-1} S_i(y) = \int_{\mathbb{I}} f(x) K_n^\alpha(y-x) d\mu(x) = (f * K_n^\alpha)(y) \quad (n \in \mathbb{N}, y \in \mathbb{I}).$$

Let $\mathcal{I}^2 := \{I^2 = I_1 \times I_2 \mid I_1, I_2 \in \mathcal{I}, |I_1| = |I_2|\}$ denote the collection of dyadic squares. Given $x = (x_1, x_2) \in \mathbb{I}^2$ and $m \in \mathbb{N}$ the dyadic square of area 2^{-2m} containing x is given by $I_m(x_1) \times I_m(x_2) =: I_m(x)$.

The dyadic maximal function of an $f \in L^1(\mathbb{I}^j)$ is defined by

$$f^*(x) = \sup_m \frac{1}{|I_m(x)|} \left| \int_{I_m(x)} f \right| \quad (x \in \mathbb{I}^j, j \in \{1, 2\}).$$

Furthermore the dyadic Hardy space is $H(\mathbb{I}^j) := \{f \in L^1(\mathbb{I}^j) \mid \|f\|_H := \|f^*\|_1 < \infty\}$.

Define the two-dimensional character system $(v_n, n \in \mathbb{N}^2)$, Dirichlet kernel functions and for $\alpha, \beta \in \mathbb{R}$ the Cesàro kernel functions on \mathbb{I}^2 as the Kronecker products of the one-dimensional functions:

$$\begin{aligned} v_n(x) &:= v_{n_1}(x_1)v_{n_2}(x_2), \quad D_n(x) := D_{n_1}(x_1)D_{n_2}(x_2), \\ K_n^{\alpha, \beta}(x) &:= K_{n_1}^\alpha(x_1)K_{n_2}^\beta(x_2) \quad (x = (x_1, x_2) \in \mathbb{I}^2, n = (n_1, n_2) \in \mathbb{N}^2). \end{aligned}$$

Now, the two-dimensional Fourier coefficients, the n -th rectangular partial sum of the Fourier series and the n -th (C, α) means of $f \in L^1(\mathbb{I}^2)$ are

$$\begin{aligned} \hat{f}(n) &:= \int_{\mathbb{I}^2} f \bar{v}_n d\mu \quad (n \in \mathbb{N}^2, y \in \mathbb{I}^2), \\ S_n f(y) &:= \sum_{k_1=0}^{n_1-1} \sum_{k_2=0}^{n_2-1} \hat{f}(k_1, k_2) v_{k_1, k_2}(y) = \int_{\mathbb{I}^2} f(x) D_n(y-x) d\mu(x) \\ \sigma_{n_1, n_2}^{\alpha, \beta} f(y) &:= \frac{1}{A_{n_1}^\alpha A_{n_2}^\beta} \sum_{k_1=0}^{n_1} \sum_{k_2=0}^{n_2} A_{n_1-k_1}^{\alpha-1} A_{n_2-k_2}^{\beta-1} S_k f(y) = \int_{\mathbb{I}^2} f(x) K_{n_1, n_2}^{\alpha, \beta}(y-x) d\mu(x). \end{aligned}$$

Note, that $\sigma_{n_1, n_2}^{\alpha, \beta} f(y) = f * (K_{n_1}^\alpha \times K_{n_2}^\beta)$, where \times denotes the Kronecker product.

Let us consider

$$L \log^+ L(\mathbb{I}^2) := \left\{ f \in L^1(\mathbb{I}^2) : \int_{\mathbb{I}^2} |f| \log^+ |f| < \infty \right\}.$$

2. Summability results for one-dimensional functions

Some historical background concerning the Cesàro summability of Fourier series with respect to:

- the trigonometric system $(\exp(2\pi i k x), k \in \mathbb{Z})$: The a.e. convergence $\sigma_n f \rightarrow f$ was showed by Lipót Fejér in 1900 (at each point of continuity) and by Henri Lebesgue in 1926 for functions $f \in L^1([0, 2\pi])$.

- the Walsh-Paley system: N.J. Fine proved that $\sigma_n^\alpha f \rightarrow f$ a.e. in 1949 in [6] for continuous functions and in 1955 in [7] for $\alpha > 0, f \in L^1(\mathbb{I})$. F. Schipp [26] gave a simple proof for the case $\alpha = 1$ and his techniques have been expanded

to obtain new results for bounded linear operators to generalize the results from Walsh-Fourier series to Walsh-Fourier-Stieltjes series.

Although the topological structure of the 2-adic group is similar to that of the dyadic group, this theorem remained open for a long time in the 2-adic case.

Results concerning the 2-adic Fejér and Cesàro means of Fourier series with respect to the 2-adic characters ($v_n, n \in \mathbb{N}$):

-In 1997 Gát[10] proved the a.e. convergence $\sigma_n^1 f \rightarrow f$ for integrable functions $f \in L^1([0,1])$ in the 2-adic case. This was a more than 20 year old conjecture of Taibleson [33], and it was proved using a decomposition of the 2-adic Dirichlet kernel due to F. Schipp and W.R. Wade. He showed that the maximal operator σ^* is of type (H^1, L^1) and of weak type $(1,1)$. In 2007, Gát[8] proved the a.e. convergence of Cesàro means $\sigma_n^\alpha f \rightarrow f$ for integrable functions f and $\alpha > 0$.

-The 2-adic additive characters ($v_n, n \in \mathbb{N}$) form a **UDMD-product system**, thus the following propositions can be seen also as special cases of the theorems concerning UDMD-product systems. See Schipp-Wade[23], pp. 90-92 and 102-106.

$$\lim_{n \rightarrow \infty} \|S_{2^n} f - f\|_q = 0 \quad (f \in L^q(\mathbb{I}), 1 \leq q < \infty), \quad (1)$$

$$\lim_{m \rightarrow \infty} \|S_m f - f\|_q = 0 \quad (f \in \mathbb{L}^q(\mathbb{I}), 1 < q < \infty), \quad (2)$$

$$\lim_{n \rightarrow \infty} \|\sigma_n f - f\|_q = 0 \quad (f \in L^1(\mathbb{I}), 1 \leq q < \infty), \quad (3)$$

$$S_{2^n} f \rightarrow f \quad a.e. \quad (f \in \mathbb{L}^1(\mathbb{I})), \quad (4)$$

$$S_m f \rightarrow f \quad a.e. \quad (f \in \mathbb{L}^q(\mathbb{I}), q > 1), \quad (5)$$

$$\sigma_n f \rightarrow f \quad a.e. \quad (f \in \mathbb{L}^1(\mathbb{I})). \quad (6)$$

Moreover, (2) and (3) holds for $q = \infty$ when f is continuous on \mathbb{I} . (4) holds a.e. and also at every point of continuity of f . (5) can be found in Schipp[25].

- Many results are known on the **Nörlund logarithmic means** with respect to the Walsh-Paley system but the problem is investigated also with respect to the 2-adic characters. Theorems concerning Nörlund logarithmic means with respect to Walsh-Paley system can be found in Goginava-Tkebuchava[16], Goginava[17], and for bounded Vilenkin systems in Gát-Nagy[14]. On 2-adic Nörlund logarithmic means

$$t_n f := \frac{1}{l_n} \sum_{k=0}^{n-1} \frac{S_k f}{n-k} \quad (n \in \mathbb{P}) \quad \text{where } l_n := \sum_{k=1}^{n-1} \frac{1}{k}$$

with respect to 2-adic character system Blahota[4] obtained: if $(m_n, n \in \mathbb{P})$ is a series of positive integers for which $\sum_{n=1}^{\infty} \frac{\log^2(m_n - 2^{\lfloor \log m_n \rfloor + 1})}{\log m_n} < \infty$, then the maximal operator $t^* f(x) := \sup_{n \in \mathbb{P}} |t_{m_n} f(x)|$ is of weak type $(1,1)$. Thus in this case $t_{m_n} f \rightarrow f$ a.e. as $n \rightarrow \infty$ ($f \in L^1(\mathbb{I})$). Thus $t_{2^n} f \rightarrow f$ a.e. as $n \rightarrow \infty$ ($f \in L^1(\mathbb{I})$).

- The **Vilenkin-like systems** introduced by Gát in [13] are a common generalizations of the 2-adic additive characters, Walsh-Paley system, the Vilenkin system, which is the m -adic character system, the UDMD-product systems and others. Many of the 2-adic results hold also with respect to Vilenkin-like systems. Boundedness theories on the Fejér kernel, convergence in $L^p(G_m)$ -norm of functions $f \in L^p(G_m)$ ($1 \leq p < \infty$) and $(C,1)$ -summability for $f \in L^p(G_m)$ can be

found in Gát[13]. I. Blahota established inequalities in [3] for weighted maximal functions of the Dirichlet and Fejér kernels, and in [2] on the maximal value of the series of Dirichlet and Fejér kernels. K. Nagy proved in [21] the (C,1) summability of integrable functions for two-dimensional Vilenkin-like systems provided that the set of indices is in a cone-like set.

3. Results for two- and more-dimensional functions

Some historical background concerning Fourier series with respect to:

- the trigonometric system: In 1939 Marcinkiewicz and Zygmund[20] proved that the Fejér means $\sigma_{n,m}^1 f$ of the trigonometric Fourier series of two variable integrable functions converge almost everywhere to the function if the ratio of the indices of the means remain in some positive cone around the identical function as they tend to infinity. (That is, $\frac{1}{\beta} \leq \frac{m}{n} \leq \beta$ for some $\beta \in \mathbb{R}$.) (The convergence with restriction on the indices.)

In 1935 Jessen, Marcinkiewicz and Zygmund[18] proved that the Fejér means $\sigma_{n,m}^1 f$ of the trigonometric Fourier series of two variable functions $f \in L \log^+ L([0, 2\pi]^2)$ converge almost everywhere to the function when $\min\{n, m\} \rightarrow \infty$. (The convergence in Pringsheim sense.)

- the Walsh-Paley system: F. Móricz, F. Schipp, and W.R. Wade [19] showed the convergence in Pringsheim sense for functions in $L \log^+ L$, and the convergence with restriction for L^1 functions, but only for $\sigma_{2^n, 2^m} f \rightarrow f$, $|n - m| \leq C$.

In 1996 Gát[9] and Weisz[34] proved independently from each other the convergence with restriction on the indices for L^1 functions for all index pairs, not only powers of two. As a generalization of this result, in 2010 Gát and Nagy [15] showed the pointwise convergence of cone-like restricted two-dimensional Cesàro means of Walsh-Fourier series.

- the characters of the 2-adic additive group: The a.e. convergence result $\sigma_n^\alpha f \rightarrow f$ ($f \in L^1(\mathbb{I})$, $\alpha > 0$) holds for two- and more-dimensional functions: In 2014 Gát and I. Simon proved the following: Let $\alpha, \beta > 0$, $f \in L \log^+ L(\mathbb{I}^2)$. Then we have a.e. convergence $\sigma_{n,m}^{\alpha,\beta} f \rightarrow f$ as $m, n \rightarrow \infty$.

Results on the **Marcinkiewicz means** are also known with respect to the 2-adic characters. Among the results with respect to the trigonometric system let us mention Zhizhiasvili[35]. With respect to the 2-adic system

$$\sigma_n^M f(x_1, x_2) := \frac{1}{n} \sum_{k=0}^{n-1} S_{k,k} f(x_1, x_2) \quad (n \in \mathbb{P}).$$

Blahota and Gát [1] obtained the following: $\sigma_n^M f(x_1, x_2) \rightarrow f(x_1, x_2)$ a.e. as $n \rightarrow \infty$ ($f \in L^1(\mathbb{I}^2)$).

4. Convergence and summability results with respect to other systems on the 2-adic group

As a general construction, let $\{\phi_n, n \in \mathbb{N}\}$ denote the character set of the studied additive group, and consider a measure-preserving variable transformation $T : \mathbb{I} \rightarrow$

ℓ. If we consider the Fourier expansion with respect to the system $(\phi_n \circ T, n \in \mathbb{N})$, then its partial sums $S_n^T f$ and Cesàro means $\sigma_n^{\alpha, T} f$ can be expressed by the original ones, that is, by partial sums and Cesàro means of Fourier series with respect to the characters.

Theorem 4.1: For any $f \in L^1(\mathbb{I})$, $n \in \mathbb{P}$ hold

$$S_n^T f = [S_n(f \circ T^{-1})] \circ T, \text{ and} \quad (7)$$

$$\sigma_n^{\alpha, T} f = [\sigma_n^\alpha(f \circ T^{-1})] \circ T, \quad (8)$$

where S_n and σ_n stand for the corresponding notions with respect to the characters $\{\phi_n, n \in \mathbb{N}\}$ of the additive group.

Therefore, the following theorem holds.

Theorem 4.2: In each point $x \in \mathbb{I}^d$, where $\lim_{n \rightarrow \infty} S_n f(x) = f(x)$ is true, the convergence $\lim_{n \rightarrow \infty} S_n^T f = f(x)$ also holds. Furthermore in each point $x \in \mathbb{I}^d$, where $\lim_{n \rightarrow \infty} \sigma_n^\alpha f(x) = f(x)$ is true, the convergence $\lim_{n \rightarrow \infty} (\sigma_n^{\alpha, T} f)(x) = f(x)$ also holds.

On the complex field similar expressions were used in terms of the scalar products for the trigonometric system in Bokor-Schipp [5]. On the dyadic and 2-adic fields the presented proposition enabled to handle a.e. convergence and summability questions of Fourier series with respect to the discrete Laguerre and $(v_n \circ \gamma, n \in \mathbb{N})$ systems in I. Simon [28] and [29]. Professor F. Schipp claimed that the proposition is true for general measure-preserving transformations.

Acknowledgment.

Research supported by project TÁMOP-4.2.2.A-11/1/KONV-2012-0051.

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