Available online at www.tjnsa.com
J. Nonlinear Sci. Appl. 9 (2016), 553–567
Research Article

Journal of Nonlinear Science and Applications
Print: ISSN 2008-1898 Online: ISSN 2008-1901

Analysis of a stochastic food chain model with finite delay

Jing Fu[a], Haihong Li[b], Qixing Han[c,*], Haixia Li[d]

[a]*School of Mathematics, Changchun Normal University, Changchun 130032, Jilin, P. R. China.*
[b]*Department of Basic Courses, Air Force Aviation University, Changchun 130022, Jilin, P. R. China.*
School of Mathematics, Jilin University, Changchun 130024, Jilin, P. R. China.
[c]*School of Mathematics, Changchun Normal University, Changchun 130032, Jilin, P. R. China.*
[d]*School of Business, Northeast Normal University, Changchun 130024, Jilin, P. R. China.*

Communicated by Yeol Je Cho

Abstract

A stochastic three species predator-prey time-delay chain model is proposed and analyzed. Sufficient conditions for persistence in time average and non-persistence are established. Numerical simulations are carried out to support our results. ©2016 All rights reserved.

Keywords: Stochastic differential equation, persistent, non-persistent.
2010 MSC: 60H05, 60G44.

1. Introduction

The most exciting modern application of mathematics is used in biology. The continuing health of mathematics and the complexity of the biological sciences make interdisciplinary involvement essential. In the past few decades, mathematical biology research has opened up a new exciting cornucopia of challenging problems for the mathematicians. On the other hand, mathematical modeling offers another research tool commensurate with new powerful laboratory techniques for the biologists.

As we know, two species systems such as predator-prey, plant-pest systems et cetera have long been one of the dominant themes in both ecology and mathematical ecology due to its universal importance. After that, the predator-prey chain model is the typical representative. To the best of our knowledge, it

*Corresponding author
Email address: hanqixing123@163.com (Qixing Han)

Received 2015-04-22

was only in the late 70s that some interest in the mathematics of tritrophic food chain models (composed of prey, predator and superpredator) emerged [5, 6]. Three-species systems like plant-herbivore-parasitoid, plant-pest-predator et cetera are emerging in different branches of biology in their own right. One of the most famous models for population dynamics is the Lotka-Volterra predator-prey system which has received plenty of attention and has been studied extensively, refer to [3, 10, 18]. Specially persistence and extinction of this model are interesting topics.

The three species predator-prey chain model is described as follows:

$$\begin{cases} \dot{x_1}(t) = x_1(t)\left(a_1 - b_{11}x_1(t) - b_{12}x_2(t)\right), \\ \dot{x_2}(t) = x_2(t)\left(-a_2 + b_{21}x_1(t) - b_{22}x_2(t) - b_{23}x_3(t)\right), \\ \dot{x_3}(t) = x_3(t)\left(-a_3 + b_{32}x_2(t) - b_{33}x_3(t)\right), \end{cases} \tag{1.1}$$

where $x_i(t)$ $(i = 1, 2, 3)$ represents the densities of prey, mid-level predator and top predator species at time t, respectively. The parameters a_1, a_2, a_3, b_{ii} $(i = 1, 2, 3)$ are positive constants that stand for intrinsic growth rate, predator death rate of the second species, predator death rate of the third species, coefficient of internal competition respectively. b_{21}, b_{32} represent saturated rate of the second and the third predator, b_{12}, b_{23} represent the decrement rate of predator to prey. System (1.1) describes an three species predator-prey chain model in which the latter preys on the former. From a biological viewpoint, we not only require the positive solution of the system but also require its unexploded property in any finite time and stability.

We know that the global asymptotic stability of a positive equilibrium $x^* = (x_1^*, x_2^*, x_3^*)$ holds and is global stability if the following condition holds:

$$a_1 - fb_{11}b_{21}a_2 - fb_{11}b_{22} + b_{12}b_{21}b_{21}b_{32}a_3 > 0,$$

which could refer to [9].

In recent times, it is well understood that many of the processes, both natural and man-made, in biology, medicine et cetera involve time-delays. Time-delays occur so often, in almost every situation, that to ignore them is to ignore reality. Kuang [17] mentioned that animals must take time to digest their food before further activities and responses take place. So, any model of species dynamics without delays is an approximation at best. Criteria for three classes of models of single-species dynamics with a single discrete delay to have a globally asymptotically stable positive equilibrium independent of the length of delay was established by Freedman and Gopalsamy [4]. By constructing appropriate liapunov functionals for the models, Ma [25] studied the global stability of volterra models with time delay. Hence, we introduce time-delays in system (1.1) and assume that the mid-level predator species need time τ to possess the ability of predation after it was born and it captures only the adult prey species with maturation time τ, while the top predator species need time τ to possess the ability of predation and it captures only adult mid-level predator species with maturation time τ ([8, 13, 20]). Then we get

$$\begin{cases} \dot{x_1}(t) = x_1(t)\left(a_1 - b_{11}x_1(t) - b_{12}x_2(t-\tau)\right), \\ \dot{x_2}(t) = x_2(t)\left(-a_2 + b_{21}x_1(t-\tau) - b_{22}x_2(t) - b_{23}x_3(t-\tau)\right), \\ \dot{x_3}(t) = x_3(t)\left(-a_3 + b_{32}x_2(t-\tau) - b_{33}x_3(t)\right). \end{cases} \tag{1.2}$$

However, population dynamics in the real world is inevitably affected by environmental noise(see, e.g. [7, 8]). Parameters involved in the system are not absolute constants, they always fluctuate around some average values. The deterministic models assume that parameters in the systems are deterministic irrespective of environmental fluctuations imposes some limitations in mathematical modeling of ecological systems. So we can not omit the influence of the noise on the system. Recently many authors have discussed population systems subject to white noise (see, e.g. [12, 14, 21]). May (see, e.g. [23]) pointed out that due to continuous fluctuation in the environment, the birth rates, death rates, saturated rate, competition coefficients and all other parameters involved in the model exhibit random fluctuation to some extent, and as a result the equilibrium population distribution never attains a steady value, but fluctuates randomly around some average value. Sometimes, large amplitude fluctuation in population will lead to the extinction of certain species, which does not happen in deterministic models.

Therefore, Lotka-Volterra predator-prey chain models in random environments are becoming more and more popular. Ji *et al.* [14, 15] investigated the asymptotic behavior of the stochastic predator-prey system with perturbation. Liu and Chen introduced periodic constant impulsive immigration of predator into predator-prey system and gave conditions for the system to be extinct and permanence. Polansky [24] and Barra *et al.* [1] have given some special systems of their invariant distribution. After that, Gard [9] analyzed that under some conditions the stochastic food chain model exists an invariant distribution. Mao and Yuan[22] have discuss non explosion, persistence, and asymptotic stability of the stochastic differential delay equations, they reveal that the noise will not only suppress a potential population explosion in the delay Lotka-Volterra model but will also make the population to be stochastically ultimately bounded. However, seldom people investigate the persistent and non-persistent of the food chain time-delay model with stochastic perturbation. I have studied the food chain model with stochastic perturbation in [19], and this paper is a continuation of the previous article.

In this paper, we introduce the white noise into the intrinsic growth rate of system (1.2), and suppose $a_i \to a_i + \sigma_i \dot{B}_i(t)$ $(i = 1, 2, 3)$, then we obtain the following stochastic system

$$\begin{cases} dx_1(t) = x_1(t)\left(a_1 - b_{11}x_1(t) - b_{12}x_2(t-\tau)\right)dt + \sigma_1 x_1(t)dB_1(t), \\ dx_2(t) = x_2(t)\left(-a_2 + b_{21}x_1(t-\tau) - b_{22}x_2(t) - b_{23}x_3(t-\tau)\right)dt - \sigma_2 x_2(t)dB_2(t), \\ dx_3(t) = x_3(t)\left(-a_3 + b_{32}x_2(t-\tau) - b_{33}x_3(t)\right)dt - \sigma_3 x_3(t)dB_3(t), \end{cases} \tag{1.3}$$

where $B_i(t)$ $(i = 1, 2, 3)$ are independent white noises with $B_i(0) = 0, \sigma_i^2 > 0$ $(i = 1, 2, 3)$ representing the intensities of the noise.

The aim of this paper is to discuss the long time behavior of system (1.3) by stochastic comparison theorem which is different from Mao and Yuan [22]. We have mentioned that $x^* = (x_1^*, x_2^*, x_3^*)$ is also the positive equilibrium of system (1.2). But, when it is suffered stochastic perturbations, there is no positive equilibrium. Hence, it is impossible that the solution of system (1.3) will tend to a fixed point. In this paper, we show that system (1.3) is persistent in time average. Furthermore, under certain conditions, we prove the population of system (1.3) will die out in probability which will not happen in deterministic system and could reveal that large white noise may lead to extinction.

The rest of this paper is organized as follows. In Section 2, we show that there is a unique non-negative solution of system (1.3). In Section 3, we show that system (1.3) is persistent in time average. While in Section 4, we consider three situations when the population of the system will be extinction. In Section 5, numerical simulations are carried out to support our results.

Throughout this paper, unless otherwise specified, let $(\Omega, \{\mathcal{F}_t\}_{t\geq 0}, P)$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions (i.e. it is right continuous and $\mathcal{F}_0$ contains all P-null sets). Let R^3_+ denote the positive cone of R^3, namely $R^3_+ = \{x \in R^3 : x_i > 0,\ 1 \leq i \leq 3\}, \bar{R}^3_+ = \{x \in R^3 : x_i \geq 0,\ 1 \leq i \leq 3\}$.

2. Existence and uniqueness of the nonnegative solution

To investigate the dynamical behavior, the first concern thing is whether the solution is global existence. Moreover, for a population model, whether the solution is nonnegative is also considered. Hence, in this section we show that the solution of system (1.3) is global and nonnegative. As we have known, in order for a stochastic differential equation to have a unique global (i.e. no explosion at a finite time) solution with any given initial value, the coefficients of the equation are generally required to satisfy the linear growth condition and local Lipschitz condition (see, e.g. [20]). It is easy to see that the coefficients of system (1.3) is locally Lipschitz continuous, so system (1.3) has a local solution. In the following we will show the global existence of this solution.

Let $N(t)$ be the solution of the non-autonomous logistic equation with random perturbation

$$dN(t) = N(t)[(a(t) - b(t)N(t))dt + \alpha(t)dB(t)], \tag{2.1}$$

where $B(t)$ is one-dimensional standard Brownian motion, $N(0) = N_0 > 0$ and N_0 is independent of $B(t)$.

Lemma 2.1 (see [16])**.** *There exists a unique continuous solution $N(t)$ of (2.1) for any initial value $N(0) = N_0 > 0$, which is global and represented by*

$$N(t) = \frac{\exp\{\int_0^t [a(s) - \frac{\sigma^2(s)}{2}]ds + \sigma(s)dB(s)\}}{1/N_0 + \int_0^t b(s)\exp\{\int_0^s [a(\tau) - \frac{\sigma^2(\tau)}{2}]d\tau + \sigma(\tau)dB(\tau)\}ds}, \quad t \geq 0. \tag{2.2}$$

In order to get the conclusion, we should introduce two systems first.

$$\begin{cases} d\Phi_1(t) = \Phi_1(t)\left(a_1 - b_{11}\Phi_1(t)\right)dt + \sigma_1\Phi_1(t)dB_1(t), \\ d\Phi_2(t) = \Phi_2(t)\left(-a_2 + b_{21}\Phi_1(t-\tau) - b_{22}\Phi_2(t)\right)dt - \sigma_2\Phi_2(t)dB_2(t), \\ d\Phi_3(t) = \Phi_3(t)\left(-a_3 + b_{32}\Phi_2(t-\tau) - b_{33}\Phi_3(t)\right)dt - \sigma_3\Phi_3(t)dB_3(t), \\ \Phi_i(t) = \xi_i(t) \in C([-\tau, 0]; R_+) \quad i = 1, 2, 3, \end{cases} \tag{2.3}$$

and

$$\begin{cases} dI_1(t) = I_1(t)\left(a_1 - b_{11}I_1(t) - b_{12}\Phi_2(t-\tau)\right)dt + \sigma_1 I_1(t)dB_1(t), \\ dI_2(t) = I_2(t)\left(-a_2 + b_{21}I_1(t-\tau) - b_{22}I_2(t) - b_{23}\Phi_3(t-\tau)\right)dt - \sigma_2 I_2(t)dB_2(t), \\ dI_3(t) = I_3(t)\left(-a_3 + b_{32}I_2(t-\tau) - b_{33}I_3(t)\right)dt - \sigma_3 I_3(t)dB_3(t), \\ I_i(t) = \xi_i(t) \in C([-\tau, 0]; R_+) \quad i = 1, 2, 3, \end{cases} \tag{2.4}$$

where

$$\Phi(t) = (\Phi_1(t), \Phi_2(t), \Phi_3(t))^{\mathsf{T}}, \;\; I(t) = (I_1(t), I_2(t), I_3(t))^{\mathsf{T}},$$

are the solutions of the above stochastic differential equations with time delay.

Theorem 2.2. *For any initial data $x(t) = \{(\xi_1(t), \xi_2(t), \xi_3(t)) : -\tau \leq t \leq 0\} \in C([-\tau, 0]; R_+^3)$, the positive solution of system (1.3) has the property that*

$$I(t) \leq x(t) \leq \Phi(t),$$

i.e.,

$$I_i(t) \leq x_i(t) \leq \Phi_i(t), \qquad i = 1, 2, 3,$$

where

$$\Phi(t) = (\Phi_1(t), \Phi_2(t), \Phi_3(t))^{\mathsf{T}}, \;\; I(t) = (I_1(t), I_2(t), I_3(t))^{\mathsf{T}},$$

are solutions of system (2.3) and (2.4).

Proof. Let $z_1(t) = \frac{1}{x_1(t)}$. Then, by Itô's formula, we have

$$\begin{aligned} dz_1(t) =& d\left(\frac{1}{x_1(t)}\right) \\ =& -\left[\left(\frac{a_1}{x_1(t)} - \frac{b_{12}x_2(t-\tau)}{x_1(t)} - b_{11}\right)dt + \frac{\sigma_1}{x_1(t)}dB_1(t)\right] + \frac{\sigma_1^2}{x_1(t)}dt \\ =& \left[(\sigma_1^2 - a_1)z_1(t) + b_{11} + \frac{b_{12}x_2(t-\tau)}{x_1(t)}\right]dt - \sigma_1 z_1(t)dB_1(t) \\ =& \left[(\sigma_1^2 - a_1)dt - \sigma_1 dB_1(t)\right]z_1(t) + \left(b_{11} + \frac{b_{12}x_2(t-\tau)}{x_1(t)}\right)dt. \end{aligned}$$

That is

$$dz_1(t) = [(\sigma_1^2 - a_1)dt - \sigma_1 dB_1(t)]z_1(t) + \left(b_{11} + \frac{b_{12}x_2(t-\tau)}{x_1(t)}\right)dt.$$

Then

$$
\begin{aligned}
z_1(t) =& e^{\int_0^t \left(\frac{\sigma_1^2}{2}-a_1\right)ds-\sigma_1 dB_1(s)} \left[\frac{1}{x_1(0)} + \int_0^t \left(b_{11} + \frac{b_{12}x_2(t-\tau)}{x_1(t)}\right) e^{\int_0^s (a_1-\frac{\sigma_1^2}{2})d\tau+\sigma_1 dB_1(\tau)} ds\right] \\
=& e^{\left(\frac{\sigma_1^2}{2}-a_1\right)t-\sigma_1 B_1(t)} \left[\frac{1}{x_1(0)} + \int_0^t \left(b_{11} + \frac{b_{12}x_2(t-\tau)}{x_1(t)}\right) e^{\left(a_1-\frac{\sigma_1^2}{2}\right)s+\sigma_1 B_1(s)} ds\right] \\
\geq& e^{\left(\frac{\sigma_1^2}{2}-a_1\right)t-\sigma_1 B_1(t)} \left[\frac{1}{x_1(0)} + \int_0^t b_{11} e^{(a_1-\frac{\sigma_1^2}{2})s+\sigma_1 B_1(s)} ds\right] \\
=& \Phi_1^{-1}(t).
\end{aligned}
$$

By Lemma 2.1, we obtain that $\Phi_1(t)$ is the solution of the following equation

$$d\Phi_1(t) = \Phi_1(t)\left(a_1 - b_{11}\Phi_1(t)\right)dt + \sigma_1\Phi_1(t)dB_1(t).$$

Hence, we have

$$x_1(t) \leq \Phi_1(t), \quad a.s..$$

On the other hand, let $z_2(t) = \frac{1}{x_2(t)}$. Then, by Itô's formula, we could derive that

$$
\begin{aligned}
dz_2(t) =& d\left(\frac{1}{x_2(t)}\right) \\
=& -\left[\left(-\frac{a_2}{x_2(t)} + \frac{b_{21}x_1(t-\tau)}{x_2(t)} - b_{22} - \frac{b_{23}x_3(t-\tau)}{x_2(t)}\right)dt - \frac{\sigma_2}{x_2(t)}dB_2(t)\right] + \frac{\sigma_2^2}{x_2(t)}dt \\
=& \left[(\sigma_2^2 + a_2)z_2(t) + b_{22} - b_{21}x_1(t-\tau)z_2(t) - b_{23}x_3(t-\tau)z_2(t)\right]dt + \sigma_2 z_2(t)dB_2(t) \\
=& \left[(\sigma_2^2 + a_2 - b_{21}x_1(t-\tau) - b_{23}x_3(t-\tau))dt + \sigma_2 dB_2(t)\right]z_2(t) + b_{22}dt,
\end{aligned}
$$

then

$$
\begin{aligned}
z_2(t) =& \frac{1}{x_2(0)} e^{\left(\frac{\sigma_2^2}{2}+a_2\right)t+\sigma_2 B_2(s)-b_{21}\int_0^t x_1(s-\tau)ds+b_{23}\int_0^t x_3(s-\tau)ds} \\
&+ b_{22}\int_0^t e^{\left(a_2+\frac{\sigma_2^2}{2}\right)(t-s)+\sigma_2(B_2(t)-B_2(s))-b_{21}\int_s^t x_1(\mu-\tau)d\mu+b_{23}\int_s^t x_3(\mu-\tau)d\mu} ds \\
\geq& \frac{1}{x_2(0)} e^{\left(\frac{\sigma_2^2}{2}+a_2\right)t+\sigma_2 B_2(s)-b_{21}\int_0^t x_1(s-\tau)ds} \\
&+ b_{22}\int_0^t e^{\left(a_2+\frac{\sigma_2^2}{2}\right)(t-s)+\sigma_2(B_2(t)-B_2(s))-b_{21}\int_s^t \Phi_1(\mu-\tau)d\mu} ds \\
=& \Phi_2^{-1}(t).
\end{aligned}
$$

Therefore

$$x_2(t) \leq \Phi_2(t), \quad a.s..$$

By Lemma 2.1, we obtain that $\Phi_2(t)$ is the solution of the following equation

$$d\Phi_2(t) = \Phi_2(t)\left(-a_2 + b_{21}\Phi_1(t-\tau) - b_{22}\Phi_2(t)\right)dt - \sigma_2\Phi_2(t)dB_2(t).$$

At last, let $z_3(t) = \frac{1}{x_3(t)}$. Then, by Itô's formula, we could derive that

$$dz_3(t) = d\left(\frac{1}{x_3(t)}\right)$$

$$
\begin{aligned}
&=-\left[\left(-\frac{a_3}{x_3(t)}+\frac{b_{32}x_2(t-\tau)}{x_3(t)}-b_{33}\right)dt-\frac{\sigma_3}{x_3(t)}dB_3(t)\right]+\frac{\sigma_3^2}{x_3(t)}dt\\
&=[(\sigma_3^2+a_3)z_2(t)+b_{33}-b_{32}x_2(t-\tau)z_3(t)]dt+\sigma_3 z_3(t)dB_3(t)\\
&=[(\sigma_3^2+a_3-b_{32}x_2(t-\tau))dt+\sigma_2 dB_3(t)]z_2(t)+b_{33}dt,
\end{aligned}
$$

then

$$
\begin{aligned}
z_3(t)=&\frac{1}{x_3(0)}e^{\left(\frac{\sigma_3^2}{2}+a_3\right)t+\sigma_3B_3(s)-b_{32}\int_0^t x_2(s-\tau)ds}\\
&+b_{33}\int_0^t e^{\left(a_3+\frac{\sigma_3^2}{2}\right)(t-s)+\sigma_3(B_3(t)-B_3(s))-b_{32}\int_s^t x_2(\mu-\tau)d\mu}ds\\
\geq&\frac{1}{x_3(0)}e^{\left(\frac{\sigma_3^2}{2}+a_3\right)t+\sigma_3B_3(s)-b_{32}\int_0^t \Phi_2(s-\tau)ds}\\
&+b_{33}\int_0^t e^{\left(a_3+\frac{\sigma_3^2}{2}\right)(t-s)+\sigma_3(B_3(t)-B_3(s))-b_{32}\int_s^t \Phi_2(\mu-\tau)d\mu}ds\\
=&\Phi_3^{-1}(t),
\end{aligned}
$$

then, it is easy to see that $\Phi_3(t)$ is the solution of the following equation

$$d\Phi_3(t)=\Phi_3(t)\left(-a_3+b_{32}\Phi_2(t-\tau)-b_{33}\Phi_3(t)\right)dt-\sigma_3\Phi_3(t)dB_3(t),$$

and

$$x_3(t)\leq\Phi_3(t),\quad a.s..$$

In the same method, we could derive that

$$x_i(t)\geq I_i(t)\ \ a.s..\ \ i=1,2,3,$$

where $I(t)=(I_1(t),I_2(t),I_3(t))^{\intercal}$ is the solution of system (2.4). □

Remark 2.3. From Lemma 2.1, we know

$$
\begin{aligned}
\frac{1}{\Phi_1(t)}&=\frac{1}{x_1(0)}e^{\left(\frac{\sigma_1^2}{2}-a_1\right)t-\sigma_1B_1(t)}+b_{11}\int_0^t e^{\left(\frac{\sigma_1^2}{2}-a_1\right)(t-s)-\sigma_1(B_1(t)-B_1(s))}ds,\\
\frac{1}{\Phi_2(t)}&=\frac{1}{x_2(0)}e^{(\frac{\sigma_2^2}{2}+a_2)t+\sigma_2B_2(t)-b_{21}\int_0^t\Phi_1(s-\tau)ds}+b_{22}\int_0^t e^{\left(\frac{\sigma_2^2}{2}+a_2\right)(t-s)+\sigma_2(B_2(t)-B_2(s))-b_{21}\int_s^t\Phi_1(\mu-\tau)d\mu}ds,\\
\frac{1}{\Phi_3(t)}&=\frac{1}{x_3(0)}e^{\left(\frac{\sigma_3^2}{2}+a_3\right)t+\sigma_3B_3(t)-b_{32}\int_0^t\Phi_2(s-\tau)ds}+b_{33}\int_0^t e^{\left(\frac{\sigma_3^2}{2}+a_3\right)(t-s)+\sigma_3(B_3(t)-B_3(s))-b_{32}\int_s^t\Phi_2(\mu-\tau)d\mu}ds;
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{1}{I_1(t)}&=\frac{1}{x_1(0)}e^{\left(\frac{\sigma_1^2}{2}-a_1\right)t-\sigma_1B_1(t)+b_{12}\int_0^t\Phi_2(s-\tau)ds}+b_{11}\int_0^t e^{\left(\frac{\sigma_1^2}{2}-a_1\right)(t-s)-\sigma_1(B_1(t)-B_1(s))+b_{12}\int_0^t\Phi_2(\mu-\tau)d\mu}ds,\\
\frac{1}{I_2(t)}&=\frac{1}{x_2(0)}e^{\left(\frac{\sigma_2^2}{2}+a_2\right)t+\sigma_2B_2(t)-b_{21}\int_0^t I_1(s-\tau)ds+b_{23}\int_0^t\Phi_3(s-\tau)ds}\\
&\quad+b_{22}\int_0^t e^{(\frac{\sigma_2^2}{2}+a_2)(t-s)+\sigma_2(B_2(t)-B_2(s))-b_{21}\int_s^t I_1(\mu-\tau)d\mu+b_{23}\int_0^t\Phi_3(\mu-\tau)d\mu}ds,
\end{aligned}
$$

$$\frac{1}{I_3(t)} = \frac{1}{x_3(0)} e^{\left(\frac{\sigma_3^2}{2}+a_3\right)t+\sigma_3 B_3(t)-b_{32}\int_0^t I_2(s-\tau)ds} + b_{33}\int_0^t e^{\left(\frac{\sigma_3^2}{2}+a_3\right)(t-s)+\sigma_3(B_3(t)-B_3(s))-b_{32}\int_s^t I_2(\mu-\tau)d\mu} ds.$$

From the representations of $\Phi_i(t)$ and $I_i(t)$, (i=1,2,3), Theorem 2.2 tells us the species will not reach zero in finite time.

From now on, we denote the unique global positive solution of system (1.3) with the given initial data $\xi = \{\xi(t) = (\xi_1(t), \xi_2(t), \xi_3(t)) : -\tau \le t \le 0\} \in C([-\tau, 0]; R_3^+)$ by $x(t, \xi)$. In the same way, we define the solutions of system (2.3) and (2.4) by $\Phi(t, \xi), I(t, \xi)$.

3. Persistent in time average

There is no equilibrium of system (1.3). Hence we can not show the permanence of the system by proving the stability of the positive equilibrium as the deterministic system. In this section we first show that this system is persistent in mean. Before we give the result, we should do some prepare work.

Chen *et al.* in [2] proposed the definition of persistence in mean for the deterministic system. Here, we also use this definition for the stochastic system.

Definition 3.1. System (1.3) is said to be persistent in mean, if

$$\liminf_{t\to\infty} \frac{1}{t}\int_0^t x_3(s)ds > 0, \quad a.s..$$

Before give the result, we do some prepare work.

Lemma 3.2 ([26])**.** *Let $f \in C([0,+\infty)\times\Omega,(0,+\infty))$, $F \in C([0,+\infty)\times\Omega, R)$. If there exist positive constants λ_0, λ, such that*

$$log f(t) \ge \lambda t - \lambda_0 \int_0^t f(s)ds + F(t), \quad t \ge 0 \quad a.s., \tag{3.1}$$

and $\lim_{t\to\infty} \frac{F(t)}{t} = 0$ a.s., then

$$\liminf_{t\to\infty} \frac{1}{t}\int_0^t f(s)ds \ge \frac{\lambda}{\lambda_0}, \quad a.s.. \tag{3.2}$$

From Lemma 3.2, it is easy to see that we could get Lemma 3.3 and Lemma 3.4 with the same method.

Lemma 3.3. *Let $f \in C([0,+\infty)\times\Omega,(0,+\infty))$, $F \in C([0,+\infty)\times\Omega, R)$. If there exist positive constants λ_0, λ, such that*

$$log f(t) \le \lambda t - \lambda_0 \int_0^t f(s)ds + F(t), \quad t \ge 0 \quad a.s., \tag{3.3}$$

and $\lim_{t\to\infty} \frac{F(t)}{t} = 0$ a.s., then

$$\limsup_{t\to\infty} \frac{1}{t}\int_0^t f(s)ds \le \frac{\lambda}{\lambda_0}, \quad a.s.. \tag{3.4}$$

Lemma 3.4. *Let $f \in C([0,+\infty)\times\Omega,(0,+\infty))$, $F \in C([0,+\infty)\times\Omega, R)$. If there exist positive constants λ_0, λ, such that*

$$log f(t) = \lambda t - \lambda_0 \int_0^t f(s)ds + F(t), \quad t \ge 0 \quad a.s., \tag{3.5}$$

and $\lim_{t\to\infty} \frac{F(t)}{t} = 0$ a.s., then

$$\lim_{t\to\infty} \frac{1}{t}\int_0^t f(s)ds = \frac{\lambda}{\lambda_0}, \quad a.s.. \tag{3.6}$$

Assumption 1.

$$r_1 - fb_{11}b_{21}r_2 - fb_{11}b_{22} + b_{12}b_{21}b_{21}b_{32}r_3 > 0, \quad r_1 = a_1 - \frac{\sigma_1^2}{2} > 0, \quad r_i = a_i + \frac{\sigma_i^2}{2} \quad i = 2, 3.$$

Lemma 3.5. *If Assumption 1 is satisfied, the the solution $\Phi(t,\xi)$ of system (2.3) has the following property:*

$$\lim_{t\to\infty} \frac{\log \Phi_i(t)}{t} = 0, \quad \lim_{t\to\infty} \frac{1}{t}\int_0^t \Phi_i(s)ds = M_i, \quad a.s., \tag{3.7}$$

where

$$M_1 = \frac{r_1}{b_{11}}, \quad M_2 = \frac{r_1 b_{21} - r_2 b_{11}}{b_{11}}, \quad M_3 = \frac{r_1 b_{21} b_{32} - r_2 b_{11} b_{32} - r_3 b_{11} b_{22}}{b_{11} b_{22} b_{33}}.$$

Proof. From the result in [15] and Assumption 1 is satisfied, we know

$$\lim_{t\to\infty} \frac{\log \Phi_1(t)}{t} = 0, \quad \lim_{t\to\infty} \frac{1}{t}\int_0^t \Phi_1(s)ds = \frac{a_1 - \frac{\sigma_1^2}{2}}{b_{11}} = \frac{r_1}{b_{11}} = M_1, \ a.s., \tag{3.8}$$

besides, according to Itô's formula, the second population of system (2.3) is changed into

$$d\log \Phi_2(t) = (-r_2 + b_{21}\Phi_1(t-\tau) - b_{22}\Phi_2(t))dt - \sigma_2 dB_2(t).$$

It then follows

$$\log \Phi_2(t) = \log \Phi_2(0) - r_2 t + b_{21}\int_0^t \Phi_1(s-\tau)ds - b_{22}\int_0^t \Phi_2(s)ds - \sigma_2 B_2(t). \tag{3.9}$$

Notice that

$$\int_0^t \Phi_1(s-\tau)ds = \int_{-\tau}^{t-\tau} \Phi_1(s)ds = \int_{-\tau}^0 \xi_1(s)ds + \int_0^t \Phi_1(s)ds - \int_{t-\tau}^t \Phi_1(s)ds, \tag{3.10}$$

and from the second equation of (3.8), we get $\lim\limits_{t\to\infty} \frac{1}{t}\int_{t-\tau}^t \Phi_1(s)ds = 0$, dividing the equation (3.10) both sides by t, and taking $t \to \infty$, yields

$$\lim_{t\to\infty} \frac{1}{t}\int_0^t \Phi_1(s-\tau)ds = \lim_{t\to\infty} \frac{1}{t}\int_0^t \Phi_1(s)ds = M_1,$$

so

$$\lim_{t\to 0} \frac{\log \Phi_2(0) - r_2 t + b_{21}\int_0^t \Phi_1(s-\tau)ds - \sigma_2 B_2(t)}{t} = -r_2 + b_{21}\frac{r_1}{b_{11}}.$$

With Lemma 3.4 and Assumption 1 we could get

$$\lim_{t\to\infty} \frac{1}{t}\int_0^t \Phi_2(s)ds = \frac{-r_2 + b_{21}\frac{r_1}{b_{11}}}{b_{22}} = \frac{r_1 b_{21} - r_2 b_{11}}{b_{11} b_{22}} = M_2 > 0. \tag{3.11}$$

Let (3.9) divide t, and $t \to \infty$, together with (3.8) and (3.10), consequently

$$\lim_{t\to\infty} \frac{\log \Phi_2(t)}{t} = 0.$$

Similarly, according to Ito's formula, the third population of system (2.3) is changed into

$$d\log \Phi_3(t) = (-r_3 + b_{32}\Phi_2(t-\tau) - b_{33}\Phi_3(t))dt - \sigma_3 dB_3(t),$$

it then follows

$$\log \Phi_3(t) = \log \Phi_3(0) - r_3 t + b_{32} \int_0^t \Phi_2(s-\tau)ds - b_{33} \int_0^t \Phi_3(s)ds - \sigma_3 B_3(t),$$

and

$$\lim_{t\to\infty} \frac{1}{t} \int_0^t \Phi_3(s)ds = \frac{-r_3 + b_{32}\frac{r_1 b_{21} - r_2 b_{11}}{b_{11} b_{22}}}{b_{33}} = M_3 > 0,, \quad \lim_{t\to\infty} \frac{\log \Phi_3(t)}{t} = 0.$$

□

From this, together with Theorem 2.2 and Lemma 3.5, the following result is obviously true.

Theorem 3.6. *If Assumption 1 is satisfied, the the solution $x(t,\xi)$ of system* (1.3) *has the following property:*

$$\limsup_{t\to\infty} \frac{\log x_i(t)}{t} \le 0, \quad i = 1,2,3. \tag{3.12}$$

Above all, we could get

Theorem 3.7. *If Assumption 1 is satisfied, the the solution $x(t,\xi)$ of system* (1.3) *has the following property:*

$$\liminf_{t\to\infty} \frac{1}{t} \int_0^t x_3(s)ds \ge \tilde{x}_3^*, \quad a.s., \tag{3.13}$$

where $\tilde{x}^* = (\tilde{x}_1^*, \tilde{x}_2^*, \tilde{x}_3^*)$ is the only nonnegative solution of the following equation,

$$\begin{cases} r_1 - b_{11}x_1 - b_{12}x_2 = 0, \\ -r_2 + b_{21}x_1 - b_{22}x_2 - b_{23}x_3 = 0, \\ -r_3 + b_{32}x_2 - b_{33}x_3 = 0. \end{cases}$$

Proof. From system (1.3), such that

$$\begin{aligned} \frac{\log x_1(t) - \log x_1(0)}{t} &= r_1 - b_{11}\frac{1}{t}\int_0^t x_1(s)ds - b_{12}\frac{1}{t}\int_0^t x_2(s-\tau)ds + \frac{\sigma_1 B_1(t)}{t} \\ &= r_1 - b_{11}\frac{1}{t}\int_0^t x_1(s)ds - b_{12}\frac{1}{t}\left(\int_{-\tau}^0 \xi_2(s)ds - \int_{t-\tau}^t x_2(s)ds\right) \\ &\quad - b_{12}\frac{1}{t}\int_0^t x_2(s)ds + \frac{\sigma_1 B_1(t)}{t}, \end{aligned}$$

Similarly,

$$\begin{aligned} \frac{\log x_2(t) - \log x_2(0)}{t} &= -r_2 + b_{21}\frac{1}{t}\int_0^t x_1(s)ds + b_{21}\left(\frac{1}{t}\int_{-\tau}^0 \xi_1(s)ds - \frac{1}{t}\int_{t-\tau}^t x_1(s)ds\right) - b_{22}\frac{1}{t}\int_0^t x_2(s)ds \\ &\quad - b_{23}\frac{1}{t}\int_0^t x_3(s)ds - b_{23}\left(\frac{1}{t}\int_{-\tau}^0 \xi_3(s)ds - \frac{1}{t}\int_{t-\tau}^t x_3(s)ds\right) - \frac{\sigma_2 B_2(t)}{t}, \end{aligned}$$

and

$$\begin{aligned} \frac{\log x_3(t) - \log x_3(0)}{t} &= - r_3 + b_{32}\frac{1}{t}\int_0^t x_2(s)ds + b_{32}\left(\frac{1}{t}\int_{-\tau}^0 \xi_2(s)ds - \frac{1}{t}\int_{t-\tau}^t x_2(s)ds\right) \\ &\quad - b_{33}\frac{1}{t}\int_0^t x_3(s)ds - \frac{\sigma_3 B_3(t)}{t}. \end{aligned}$$

Hence

$$\begin{aligned}&\frac{c_1(\log x_1(t)-\log x_1(0))+c_2(\log x_2(t)-\log x_2(0))+c_3(\log x_3(t)-\log x_3(0))}{t}\\&=\Bigg[(r_1c_1-r_2c_2-r_3c_3)+(-b_{11}c_1+b_{21}c_2)\frac{1}{t}\int_0^t x_1(s)ds\\&+(-b_{12}c_1-b_{22}c_2+b_{32}c_3)\frac{1}{t}\int_0^t x_2(s)ds-(b_{23}c_2+b_{33}c_3)\frac{1}{t}\int_0^t x_3(s)ds\\&-c_1b_{12}\frac{1}{t}\left(\int_{-\tau}^0\xi_2(s)ds-\int_{t-\tau}^t x_2(s)ds\right)+c_2b_{21}\left(\frac{1}{t}\int_{-\tau}^0\xi_1(s)ds-\frac{1}{t}\int_{t-\tau}^t x_1(s)ds\right)\\&-c_2b_{23}\left(\frac{1}{t}\int_{-\tau}^0\xi_3(s)ds-\frac{1}{t}\int_{t-\tau}^t x_3(s)ds\right)+c_3b_{32}\left(\frac{1}{t}\int_{-\tau}^0\xi_2(s)ds-\frac{1}{t}\int_{t-\tau}^t x_2(s)ds\right)\\&\frac{c_1\sigma_1B_1(t)}{t}-\frac{c_2\sigma_2B_2(t)}{t}-\frac{c_3\sigma_3B_3(t)}{t}\Bigg].\end{aligned} \tag{3.14}$$

From Theorem 2.2, we get

$$x_i(t)\le\Phi_i(t)\ \ (i=1,2,3),$$

then

$$\lim_{t\to\infty}\frac{1}{t}\int_{t-\tau}^t x_i(s)ds=0. \tag{3.15}$$

Let $c_1=b_{21},\ c_2=b_{11},\ c_3=fb_{11}b_{22}+b_{12}b_{21}b_{32}$, together with Assumption 1, we know

$$r_1c_1-r_2c_2-r_3c_3>0.$$

According to Theorem 3.6 and equation (3.15), together with $\lim\limits_{t\to\infty}\frac{B_i(t)}{t}=0,\ (i=1,2,3)$, we could get

$$\begin{aligned}&\limsup_{t\to\infty}\frac{c_1(\log x_1(t)-\log x_1(0))+c_2(\log x_2(t)-\log x_2(0))+c_3(\log x_3(t)-\log x_3(0))}{t}\\&\qquad=(r_1c_1-r_2c_2-r_3c_3)-(c_2b_{23}+c_3b_{33})\liminf_{t\to\infty}\frac{1}{t}\int_0^t x_3(s)ds\le 0.\end{aligned}$$

Such that,

$$\liminf_{t\to\infty}\frac{1}{t}\int_0^t x_3(s)ds\ge\frac{r_1c_1-r_2c_2-r_3c_3}{c_2b_{23}+c_3b_{33}}=\tilde{x}_3^*,$$

where $\tilde{x}^*=(\tilde{x}_1^*,\tilde{x}_2^*,\tilde{x}_3^*)$ is the only nonnegative solution of the following equation when Assumption 1 is satisfied,

$$\begin{cases}r_1-b_{11}x_1-b_{12}x_2=0,\\-r_2+b_{21}x_1-b_{22}x_2-b_{23}x_3=0,\\-r_3+b_{32}x_2-b_{33}x_3=0.\end{cases}$$

□

4. Non-persistence

In the previous section, we show the solution $x(t,\xi)$ of system (1.3) is stable in time average, and in this section, we discuss the dynamics of system (1.3) when the white noise is getting larger. we show the situation when the population of system (1.3) will be non-persistent of the white noise is large, which does not happen in the deterministic system in three cases.

Definition 4.1. System (1.3) is said to be non-persistent, if there are positive constants $c_i, (i = 1, 2, 3)$ such that

$$\lim_{t\to\infty} \prod_{i=1}^{3} x_i^{c_i}(t) = 0 \ \ a.s..$$

Now we present conditions for all species or some species of (1.3) to be extinct. Consider the case

case (i): $r_1 < 0$.

According to Itô's formula, the first population of system (2.3) is changed into

$$d\log\Phi_1(t) \le (r_1 - b_{11}\Phi_1(t))dt - \sigma_1 dB_1(t).$$

If $r_1 < 0$, we could get

$$\limsup_{t\to\infty} \frac{\log\Phi_1(t)}{t} = r_1 < 0 \qquad a.s.,$$

from the stochastic comparison theorem, we have

$$\limsup_{t\to\infty} \frac{\log x_1(t)}{t} < 0 \qquad a.s.,$$

hence

$$\lim_{t\to\infty} x_1(t) = 0, \qquad a.s..$$

From the second population of system (2.3) and equation (3.10), we have

$$\frac{\log\Phi_2(t) - \log\Phi_2(0)}{t} \le -r_2 + b_{21}\frac{1}{t}\int_0^t \Phi_1(s-\tau)ds - \frac{\sigma_2 dB_2(t)}{t} \qquad a.s., \tag{4.1}$$

hence

$$\begin{aligned}\limsup_{t\to\infty} \frac{\log\Phi_2(t)}{t} &\le -r_2 + b_{21}\limsup_{t\to\infty}\frac{1}{t}\int_0^t \Phi_1(s-\tau)ds \\ &= -r_2 + b_{21}\limsup_{t\to\infty}\frac{1}{t}\int_0^t \Phi_1(s)ds = -r_2 \le 0 \qquad a.s..\end{aligned}$$

Similarly,

$$\limsup_{t\to\infty} \frac{\log\Phi_3(t)}{t} = -r_3 \le 0 \qquad a.s.,$$

and

$$\lim_{t\to\infty} x_i(t) = 0 \qquad a.s.. \qquad i = 2, 3.$$

case (ii): $r_1 > 0,\ r_1 - \frac{b_{11}}{b_{21}} r_2 < 0$.

It is clear that from the equation (4.1) and (3.8), we get

$$\limsup_{t\to\infty} \frac{\log\Phi_2(t)}{t} \le -r_2 + b_{21}\frac{r_1}{b_{11}} < 0 \qquad a.s..$$

Similarly

$$\limsup_{t\to\infty} \frac{\log\Phi_3(t)}{t} \le -r_3 + b_{32}\limsup_{t\to\infty}\frac{1}{t}\int_0^t \Phi_2(s)ds = -r_3 < 0 \qquad a.s.,$$

thus,

$$\lim_{t\to\infty} x_i(t) = 0 \qquad a.s., \qquad i = 2, 3.$$

Above all, and from the conclusion in [13], we could easily know that the distribution of $x_1(t)$ converges weekly to the probability measure with density

$$f^*(\zeta) = C_0 \zeta^{2r_1/\sigma_1^2 - 1} e^{-2b_{11}\zeta/\sigma_1^2},$$

where $C_0 = (2b_{11}/\sigma_1^2)^{2r_1/\sigma_1^2}/\Gamma(2r_1/\sigma_1^2)$, and

$$\lim_{t\to\infty} \frac{1}{t}\int_0^t x_1(s)ds = \frac{r_1}{b_{11}}, \quad a.s..$$

case (iii): $r_1 - fb_{11}b_{21}r_2 - fb_{11}b_{22} + b_{12}b_{21}b_{21}b_{32}r_3 < 0$.

It is clear that from (3.14), let $c_1 = b_{21},\ c_2 = b_{11},\ c_3 = fb_{11}b_{22} + b_{12}b_{21}b_{32}$, with $\lim_{t\to\infty} \frac{B_i(t)}{t} = 0, i = 1, 2, 3$, we get

$$\begin{aligned}
&\limsup_{t\to\infty} \frac{c_1(\log x_1(t) - \log x_1(0)) + c_2(\log x_2(t) - \log x_2(0)) + c_3(\log x_3(t) - \log x_3(0))}{t} \\
&= (r_1c_1 - r_2c_2 - r_3c_3) - (c_2b_{23} + c_3b_{33}) \liminf_{t\to\infty} \frac{1}{t}\int_0^t x_3(s)ds \\
&\le r_1c_1 - r_2c_2 - r_3c_3,
\end{aligned} \tag{4.2}$$

moreover,

$$\limsup_{t\to\infty} \frac{\log x_1^{c_1}(t)x_2^{c_2}(t)x_3^{c_3}(t)}{t} \le r_1c_1 - r_2c_2 - r_3c_3 < 0,$$

then

$$\lim_{t\to\infty} x_1^{c_1}(t)x_2^{c_2}(t)x_3^{c_3}(t) = 0 \ \ a.s..$$

Therefore, by the above arguments, we get the follow conclusion.

Theorem 4.2. *Let $x(t,\xi)$ be the solution of system* (1.3)*, the following conclusion is founded:*

(1) *If $r_1 < 0$, then*

$$\lim_{t\to\infty} x_i(t) = 0 \quad a.s., \quad i = 1, 2, 3 \tag{4.3}$$

(2) *If $r_1 > 0,\ r_1 - \frac{b_{11}}{b_{21}} r_2 < 0$, then*

$$\lim_{t\to\infty} x_i(t) = 0 \quad a.s., \quad i = 2, 3, \tag{4.4}$$

and the distribution of $x_1(t)$ converges weekly to the probability measure with density

$$f^*(\zeta) = C_0 \zeta^{2r_1/\sigma_1^2 - 1} e^{-2b_{11}\zeta/\sigma_1^2},$$

where $C_0 = (2b_{11}/\sigma_1^2)^{2r_1/\sigma_1^2}/\Gamma(2r_1/\sigma_1^2)$, and

$$\lim_{t\to\infty} \frac{1}{t}\int_0^t x_1(s)ds = \frac{r_1}{b_{11}}, \ \ a.s.$$

(3) *If $r_1 - fb_{11}b_{21}r_2 - fb_{11}b_{22} + b_{12}b_{21}b_{21}b_{32}r_3 < 0$, then*

$$\lim_{t\to\infty} x_1^{c_1}(t)x_2^{c_2}(t)x_3^{c_3}(t) = 0, \ \ a.s., \tag{4.5}$$

where $c_1 = b_{21},\ c_2 = b_{11},\ c_3 = fb_{11}b_{22} + b_{12}b_{21}b_{32}$.

That is to say, the large white noise will lead to the population system non-persistent.

5. Numerical simulation

In this section, we give out the numerical experiment to support our results. Consider the equation

$$\begin{cases} dx_1(t) = x_1(t)\left(a_1 - b_{11}x_1(t) - b_{12}x_2(t-\tau)\right)dt + \sigma_1 x_1(t)dB_1(t), \\ dx_2(t) = x_2(t)\left(-a_2 + b_{21}x_1(t-\tau) - b_{22}x_2(t) - b_{23}x_3(t-\tau)\right)dt - \sigma_2 x_2(t)dB_2(t), \\ dx_3(t) = x_3(t)\left(-a_3 + b_{32}x_2(t-\tau) - b_{33}x_3(t)\right)dt - \sigma_3 x_3(t)dB_3(t). \end{cases} \tag{5.1}$$

By the method in [11], we have the difference equation

$$\begin{cases} x_{1,k+1} = x_{1,k} + x_{1,k}\left[(a_1 - b_{11}x_{1,k} - b_{12}x_{2,k-m})\Delta t + \sigma_1\epsilon_{1,k}\sqrt{\Delta}t + \frac{\sigma_1^2}{2}(\epsilon_{1,k}^2\Delta t - \Delta t)\right], \\ x_{2,k+1} = x_{2,k} + x_{2,k}\left[(-a_2 + b_{21}x_{1,k-m} - b_{22}x_{2,k} - b_{23}x_{3,k-m})\Delta t - \sigma_2\epsilon_{2,k}\sqrt{\Delta}t + \frac{\sigma_2^2}{2}(\epsilon_{2,k}^2\Delta t - \Delta t)\right], \\ x_{3,k+1} = x_{3,k} + x_{3,k}\left[(-a_3 + b_{32}x_{2,k-m} - b_{33}x_{3,k})\Delta t - \sigma_3\epsilon_{3,k}\sqrt{\Delta}t + \frac{\sigma_3^2}{2}(\epsilon_{3,k}^2\Delta t - \Delta t)\right], \end{cases}$$

where $\epsilon_{1,k}, \epsilon_{2,k}$ and $\epsilon_{3,k}, i = 1, 2, 3$ are the Gaussian random variables $N(0, 1)$, $r_1 = a_1 - \frac{\sigma_1^2}{2} > 0$, $r_i = a_i + \frac{\sigma_i^2}{2}$ $(i = 2, 3)$, m represents the integer part $\tau/\triangle t - 1$. Choose $(x_1(0), x_2(0), x_3(0)) \in R_+^3$, $t \in [-\tau, 0]$, and suitable parameters, by Matlab, we get Figure 1 , Figure 2 and Figure 3.

In Figure 1, when the noise is small, choose parameters satisfying the condition of Theorem 3.7, the solution of system (1.3) will persist in time average.

In Figure 2, we observe case (iii) in Theorem 4.2 and choose parameters $r_1 > 0$, $r_1 - \frac{b_{11}}{b_{21}}r_2 < 0$. As Theorem 4.2 indicated that two predators will die out in probability. The prey solution of system (1.3) will persist in time average.

In Figure 3, we observe case (i) in Theorem 4.2 and choose parameters $r_1 < 0$. As Theorem 4.2 indicated that not only predators but also prey will die out in probability when the noise of the prey is large, and it does not happen in the deterministic system. These simulated results are consistent with our theorems.

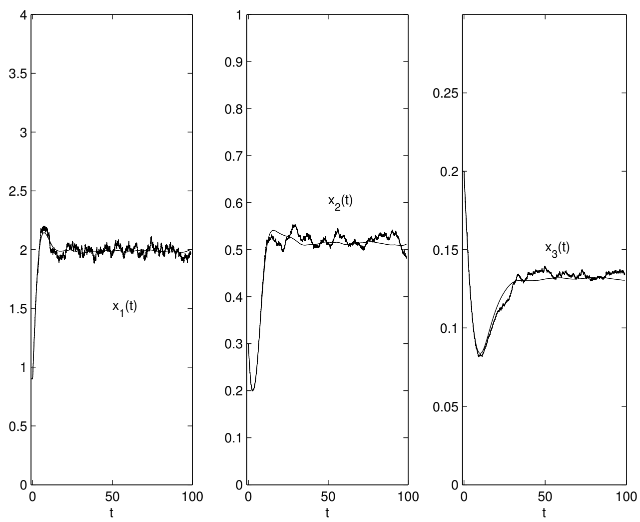

Figure 1: The solution of system (1.2) and system (1.3) with $(x_1(0), x_2(0), x_3(0)) = (0.9, 0.3, 0.2)$, $t \in [-\tau, 0]$, $a_1 = 0.7$, $a_2 = 0.3$, $a_3 = 0.1$, $b_{11} = 0.3$, $b_{12} = 0.2$, $b_{21} = 0.3$, $b_{22} = 0.5$, $b_{23} = 0.3$, $b_{32} = 0.4, b_{33} = 0.8$. The blue lines represent the solution of system (1.2), while the red lines represents the solution of system (1.3) with $\sigma_1 = 0.02, \sigma_2 = 0.01, \sigma_3 = 0.01$.

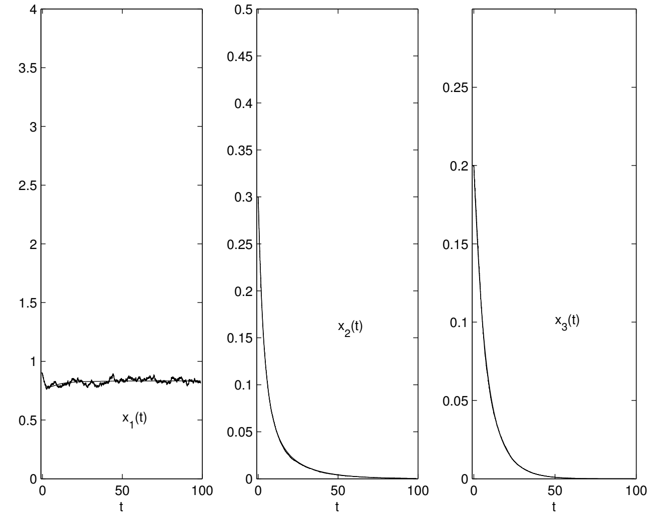

Figure 2: Two of the species will die out in probability. The solution of system (1.2) and system (1.3) with $(x_1(0), x_2(0), x_3(0)) = (0.9, 0.3, 0.2), t \in [-\tau, 0]$, $a_1 = 0.5,\ a_2 = 0.3,\ a_3 = 0.1,\ b_{11} = 0.6,\ b_{12} = 0.2,\ b_{21} = 0.3,\ b_{22} = 0.5,\ b_{23} = 0.3,\ b_{32} = 0.4, b_{33} = 0.8$. The blue lines represent the solution of system (1.2), while the red lines represents the solution of system (1.3) with $\sigma_1 = 0.02, \sigma_2 = 0.01, \sigma_3 = 0.01$.

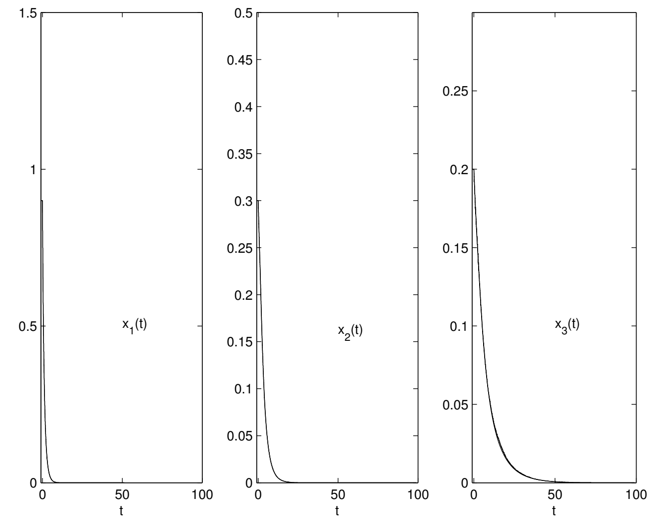

Figure 3: One of the species or both species will die out in probability. The solution of system (1.2) and system (1.3) with $(x_1(0), x_2(0), x_3(0)) = (0.9, 0.3, 0.2),\ t \in [-\tau, 0]$, $a_1 = -0.7\ a_2 = 0.3,\ a_3 = 0.1,\ b_{11} = 0.3,\ b_{12} = 0.2,\ b_{21} = 0.3,\ b_{22} = 0.5,\ b_{23} = 0.3,\ b_{32} = 0.4, b_{33} = 0.8$. The blue lines represent the solution of system (1.2), while the red lines represents the solution of system (1.3) with $\sigma_1 = 0.02, \sigma_2 = 0.01, \sigma_3 = 0.01$.

References

[1] M. Barra, D. G. Grosso, A. Gerardi, G. Koch, F. Marchetti, *Some basic properties of stochastic population models*, Springer, Berlin-New York, (1979), 155–164. 1
[2] L. S. Chen, J. Chen, *Nonlinear biological dynamical system*, Science Press, Beijing, (1993). 3
[3] H. I. Freedman, *Deterministic mathematical models in population ecology*, Marcel Dekker, New Work, (1980). 1
[4] H. I. Freedman, K. Gopalsamy, *Global stability in time delayed single species dynamics*, Bull. Math. Biol., **48** (1986), 485–492. 1
[5] H. I. Freedman, J. So, *Global stability and persistence of simple food chains*, Math. Biosci., **76** (1985), 69–86. 1
[6] H. I. Freedman, P. Waltman, *Mathematical analysis of some three-species food-chain models*, Math. Biosci., **33** (1977), 257–276. 1

[7] T. Gard, *Persistence in stochastic food web model*, Bull. Math. Biol., **46** (1984), 357–370. 1
[8] T. Gard, *Stability for multispecies population models in random environments*, Nolinear Anal., **10** (1986), 1411–1419. 1, 1
[9] T. Gard, *Introduction to stochastic differential equations*, Marcel Dekker, New Work, (1988). 1, 1
[10] K. Gopalsamy, *Global asymptotic stability in a periodic Lotka-Volterra system*, J. Austral. Math. Soc. ser. B, **27** (1985), 66–72. 1
[11] D. J. Higham, *An algorithmic introduction to numerical simulation of stochastic differential equations*, SIAM Rev., **43** (2001), 525–546. 5
[12] Y. Z. Hu, F. K. Wu, *Stochastic Lotka-Volterra model with multiple delays*, J. Math. Anal. Appl., **375** (2011), 42–57. 1
[13] C. Y. Ji, D. Q. Jiang, *Dynamics of a stochastic density dependent predator-prey system with Beddington-DeAngelis functional response*, J. Math. Anal. Appl., **381** (2011), 441–453. 1, 4
[14] C. Y. Ji, D. Q. Jiang, X. Y. Li, *Qualitative analysis of a stochastic ratio-dependent predator-prey system*, J. Comput. Appl. Math., **235** (2011), 1326–1341. 1
[15] C. Y. Ji, D. Q. Jiang, N. Z. Shi, *Analysis of a predator-prey model with modified Leslie-Gower and Holling-type II schemes with stochastic perturbation*, J. Math. Anal. Appl., **359** (2009), 482–498. 1, 3
[16] D. Q. Jiang, N. Z. Shi, *A note on nonautonomous logistic equation with random perturbation*, J. Math. Anal. Appl., **303** (2005), 164–172. 2.1
[17] Y. Kuang, *Delay differential equations with applications in population dynamics*, Academic Press, New York, (1993). 1
[18] Y. Kuang, H. L. Smith, *Global stability for infinite delay Lotka-Volterra type systems*, J. Differential Equations., **103** (1993), 221–246. 1
[19] H. H. Li, F. Z. Cong, D. Q. Jiang, H. T. Hua, *Persistence and non-persistence of a food chain model with stochastic perturbation*, Abst. Appl. Anal., **2013** (2013), 9 pages. 1
[20] X. Mao, *Stochastic differential equations and applications*, Horwood, Chichester, (1997). 1, 2
[21] X. Mao, *Delay population dynamics and environment noise*, Stoch. Dyn., **52** (2005), 149–162. 1
[22] X. Mao, C. G. Yuan, J. Zou, *Stochastic differential delay equations of population dynamics*, J. Math. Anal. Appl., **304** (2005), 296–320. 1, 1
[23] R. M. May, *Stability and complexity in model ecosystem*, Princeton University Press, New Jersey, (2001). 1
[24] P. Polansky, *Invariant distribution for multipopulation models in random environments*, Theoret. Population Biol., **16** (1979), 25–34. 1
[25] X. Q. Wen, Z. E. Ma, H. I. Freedman,*Global stability of Volterra models with time delay*, J. Math. Anal. Appl., **160** (1991), 51–59. 1
[26] P. Y. Xia, X. K. Zheng, D. Q. Jiang, *Persistence and nonpersistence of a nonautonomous stochastic mutualism system*, Abstr. Appl. Anal., **2013** (2013), 13 pages. 3.2