SOLUTION OF FRACTIONAL OXYGEN DIFFUSION PROBLEM HAVING WITHOUT SINGULAR KERNEL

BADR S ALKAHTANI, OBAID J. ALGAHTANI, RAVI SHANKER DUBEY, AND PRANAY GOSWAMI

Abstract. In the present paper, we use an efficient approach to solve fractional differential equation, Oxygen diffusion problem which is used to describe oxygen absorption in human body. The Oxygen diffusion problem is considered in new Caputo derivative of fractional order in this paper. Using an iterative approach, we derive the solutions of the modified system.

1. introduction

The distribution of oxygen into absorbing tissue was first studied by Crank and Gupta [1]. When the oxygen is allowed to diffuse into a medium, some part of the oxygen is absorbed by the medium and absorption of oxygen at the surface of the medium is maintained constant. This phase of the problem continues until a steady state is reached in which the oxygen does not penetrate any further is sealed so that no oxygen passes in or out, the medium continues to absorb the available oxygen already in it and, as a consequence, the boundary in the steady state starts to recede towards the sealed surface. Crank and Gupta [2] also employed and uniform space grid moving with the boundary and necessary interpolations are performed with either cube splines or polynomials. In this direction Noble suggested the repeated spatial subdivision [3], the heat balance integral method defined by Reynolds and Dalton [4], an orthogonal collocation for solving the partial differential equation of the diffusion of oxygen in absorbing tissue described by Liapis et al. [5]. Two numerical methods for solving the oxygen diffusion problem were proposed by Gülkaç [6]. Mitchell studied the accurate application of the integral method [7]. For more references see [8-17].
In applied mathematics, one of the most used concepts is derivative. Derivative shows the rate of change of the function. This is also helpful to describe many real phenomena. After this research, the mathematician faced some complex problems of real world to solve

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them mathematician introduce fractional derivative (see [9-13]).The concept of fractional calculus having the great importance in many branches and also important for modeling real world problem (see [14-17]).
Due to this region a lot of research work, conference, and paper publication have been done by many researchers. In this concern varies definitions of fractional derivative have been given till now. Recently the researcher describes the new fractional derivative operator named Caputo-Fabrizio fractional derivative [18-21].

1.1. **The Caputo and Fabrizio Fractional Order Derivative:** Singularity at the end point of the interval is the main problem which is faced with the definition of fractional order derivative. To avoid this problem, Caputo and Fabrizio recently proposed a new fractional order derivative which does not have any singularity. The definition is based on the convolution of a first order derivative and the exponential function, given in the following definition:
Definition 1: Let $f \in H^1(a,b)$, $b > a$, $\alpha \in [0,1]$ then the new fractional order Caputo derivative is defined as:

$$D_t^{\alpha}\left(\mathrm{f(t)}\right) = \frac{M(\alpha)}{(1-\alpha)} \int_a^t \mathrm{f}'(x)\, e^{\left[-\alpha \frac{t-x}{1-\alpha}\right]}\, \mathrm{dx}. \tag{1}$$

Here $M(\alpha)$ denote the normalization function such as $M(0) = M(1) = 1$ for detail see [18]. If $f \notin H^1(a,b)$, then the derivative can be written as

$$D_t^{\alpha}\left(\mathrm{f(t)}\right) = \frac{\alpha M(\alpha)}{(1-\alpha)} \int_a^t \left(\mathrm{f}(t) - \mathrm{f}(x)\right) e^{\left[-\alpha \frac{t-x}{1-\alpha}\right]}\, \mathrm{dx}. \tag{2}$$

Remark 1: The authors state that, if $\sigma = \frac{1-\alpha}{\alpha} \in [0,\infty)$, $\alpha = \frac{1}{1+\sigma} \in [0,1]$,then equation (2) reduces to

$$D_t^{\alpha}\left(\mathrm{f(t)}\right) = \frac{N(\sigma)}{\sigma} \int_a^t \mathrm{f}'(x)\, e^{\left[-\frac{t-x}{\sigma}\right]}\, \mathrm{dx},\ N(0) = N(\infty) = 1 \tag{3}$$

in addition

$$\lim_{\sigma \to 0} \frac{1}{\sigma} e^{\left[-\frac{t-x}{1-\alpha}\right]} = \delta(x-t). \tag{4}$$

As we have define above a new derivative, then there should be its anti-derivative, the integral of this new fractional derivative is given by Losada and Nieto [19].
Definition 2: The fractional integral of order α $(0 < \alpha < 1)$,of the function f is defined bellow:

$$I_\alpha^t\left(f(t)\right) = \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)} f(t) + \frac{2\alpha}{(2-\alpha)M(\alpha)} \int_0^t f(s)\, ds,\ t \geq 0. \tag{5}$$

Remark 2: It is clear from equation (5), the fractional integral of order α $(0 < \alpha < 1)$,is an average of function f and its integral of order 1. Hence we get the condition [19]

$$\frac{2(1-\alpha)}{(2-\alpha)M(\alpha)} + \frac{2\alpha}{(2-\alpha)M(\alpha)} = 1, \tag{6}$$

the above terms yields an explicit formula,

$$M(\alpha) = \frac{2}{(2-\alpha)}, \, 0 \le \alpha \le 1. \tag{7}$$

Due to the above relation, Nieto and Losada [19] anticipated that the new Caputo derivative of order $0 < \alpha < 1$ can be written as:

$${}_{0}^{CF}D_t^{\alpha}\,(\mathrm{f(t)}) = \frac{1}{(1-\alpha)} \int_a^t \mathrm{f}'(x) e^{\left[-\alpha \frac{t-x}{1-\alpha}\right]} \, \mathrm{dx} \tag{8}$$

Theorem 1.1. *The function $f(t)$is defined such as, for the new Caputo fractional order derivative:*

$$f^{(s)}(a) = 0, s = 1, 2, ..., n \tag{9}$$

then, we have

$$D_t^{\alpha}\left(D_t^{n}\left(f\left(t\right)\right)\right) = D_t^{n}\left(D_t^{\alpha}\left(f\left(t\right)\right)\right) \tag{10}$$

For more detail see [18,19].

1.2. **Laplace Transform.** One of the simplest and most important integral transforms which has been a subject of wide and extensive study by various authors due to its various uses in applied mathematics, is well-known Laplace transform defined as follows:

$$L(f(t)) = F(s) \tag{11}$$

$$\mathrm{L}\{\mathrm{f(t)}\,;s\} = \int_0^{\infty} \mathrm{e}^{-st}\,\mathrm{f(t)}\,\mathrm{dt}, \tag{12}$$

The Laplace transform and the Caputo-Fabrizio fractional order derivative is given as [12], defined bellow

$$L\left(\left({}_{0}^{CF}\mathrm{D}_{\mathrm{t}}^{\alpha}\right)(f(t))\right) = \left[\frac{sL(f(t)) - f(0)}{s + \alpha(1-s)}\right]. \tag{13}$$

1.3. **Oxygen Diffusion Problem Fractional Module.** The model of oxygen diffusion problem is given by Crank and Gupta [1]. The oxygen diffusion problem having two mathematical stages. At the first stage, the stable condition occurs once the oxygen is injected into either from the inside or outside of the cell then the cell surface is isolated.

At the second stage, tissues start to absorb the injected oxygen. The moving boundary problem caused by this level. The aim of this process is to find a balance position and to determine the time-dependent

moving boundary position. For detail of time-fractional of oxygen diffusion problem (see [1, 7]).
We consider the following oxygen diffusion problem:

$$\left({}_0^{CF}\mathrm{D}_{\mathrm{t}}^{\alpha}\right)(c(x,t)) = c_{xx} - 1; \;\; x,t \in \phi \tag{14}$$

with the following initial and boundary conditions

$$c(x,0) = \frac{(1-x)^2}{2}, \;\; 0 \le x \le 1, \tag{15}$$

$$\frac{\partial c}{\partial x} = 0, \; x = 0, t \ge 0 \tag{16}$$

$$c = \frac{\partial c}{\partial x} = 0, \; x = s(t), t \ge 0, with \; s(0) = 1\,. \tag{17}$$

where $0 < \alpha \le 1$.

2. Existence of the Coupled solutions:

By using the Fixed-Point theorem, we define the existence of the coupled-solution. Now first of all transform equation (14) in to an integral equation as follows:

$$c(x,t) - c(x,0) = {}_0^{CF}I_t^{\alpha}\left[c_{xx} - 1\right] \tag{18}$$

on using the definition defined by Nieto, we get

$$c(x,t) = c(x,0) + \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}\left\{\frac{\partial^2 c\,(x,t)}{\partial x^2} - 1\right\} + \frac{2\alpha}{(2-\alpha)M(\alpha)}\int_0^t \left[\frac{\partial^2 c\,(x,s)}{\partial x^2} - 1\right] ds. \tag{19}$$

Let us consider the following kernels:

$$K_1(x,t,c) = \frac{\partial^2 c\,(x,t)}{\partial x^2} - 1\,, \tag{20}$$

Theorem 2.1. *Show that K_1 satisfy Lipschiz condition and contraction if the following inequality holds:*

$$0 \le \delta^2 \le 1,$$

Proof. First of all we prove this condition forK_1. Let $c$and c_1 be two functions, then we have

$$\|K_1(x,t,c) - K_1(x,t,c_1)\| = \left\|\frac{\partial^2 c\,(x,t)}{\partial x^2} - \frac{\partial^2 c_1\,(x,t)}{\partial x^2}\right\|, \tag{21}$$

Since we know that the operator derivative satisfies the Lipchitz condition, then we can find positive parameterδsuch that:

$$\left\|\frac{\partial^2 c\,(x,t)}{\partial x^2} - \frac{\partial^2 c_1\,(x,t)}{\partial x^2}\right\| \le \delta^2 \left\|(c\,(x,t) - c_1\,(x,t))\right\|, \tag{22}$$

Putting the value from eqn. (??) into eqn. (??), we obtain:

$$\|K_1(x,t,c) - K_1(x,t,c_1)\| \leq \delta^2 \|(c(x,t) - c_1(x,t))\|, \tag{23}$$

or
where consider $\delta^2 = A$,then we get

$$\|K_1(x,t,c) - K_1(x,t,c_1)\| \leq A \|(c(x,t) - c_1(x,t))\|, \tag{24}$$

Therefore K_1satisfies the Lipschiz conditions and if in addition $0 \leq \delta^2 \leq 1$,then it is also a contraction.
We consider the following recursive formula

$$c_n(x,t) = c(x,0) + \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)} K_1(x,t,c_{n-1}) + \frac{2\alpha}{(2-\alpha)M(\alpha)} \int_0^t \{K_1(x,t,c_{n-1})\}\, ds. \tag{25}$$

With initial component

$$c_0(x,t) = c(x,0).$$

Now the difference between the consecutive terms is

$$\begin{aligned} U_n(t) = c_n(x,t) - c_{n-1}(x,t) = \tfrac{2(1-\alpha)}{(2-\alpha)M(\alpha)} K_1(x,t,c_{n-1}) - \tfrac{2(1-\alpha)}{(2-\alpha)M(\alpha)} K_1(x,t,c_{n-2}) \\ + \tfrac{2\alpha}{(2-\alpha)M(\alpha)} \int_0^t \{K_1(x,s,c_{n-1}) - K_1(x,s,c_{n-1})\}\, ds, \end{aligned} \tag{26}$$

here

$$c_n(x,t) = \sum_{i=0}^{\infty} U_n(x,t).$$

Now take norm on both sides of equation (26),we get

$$\begin{aligned} \|U_n(t)\| = \|c_n(x,t) - c_{n-1}(x,t)\| = \\ \left\| \begin{aligned} \tfrac{2(1-\alpha)}{(2-\alpha)M(\alpha)} K_1(x,s,c_{n-1}) - \tfrac{2(1-\alpha)}{(2-\alpha)M(\alpha)} K_1(x,t,c_{n-2}) \\ + \tfrac{2\alpha}{(2-\alpha)M(\alpha)} \int_0^t \{K_1(x,s,c_{n-1}) - K_1(x,s,c_{n-1})\}\, ds \end{aligned} \right\|, \end{aligned} \tag{27}$$

From the eqn. (27) we can say that

$$\begin{aligned} \|U_n(x,t)\| &= \|c_n(x,t) - c_{n-1}(x,t)\| \\ &\leq \tfrac{2(1-\alpha)}{(2-\alpha)M(\alpha)} \|K_1(x,s,c_{n-1}) - K_1(x,t,c_{n-2})\| \\ &\quad + \tfrac{2\alpha}{(2-\alpha)M(\alpha)} \left\| \int_0^t \{K_1(x,s,c_{n-1}) - K_1(x,t,c_{n-2})\} \right\| ds \end{aligned} \tag{28}$$

Since by the above discussion we have seen that kernel satisfies the Lipchitz condition, so we get:

$$\begin{aligned} &\|c_n(x,t) - c_{n-1}(x,t)\| \\ &\leq \tfrac{2(1-\alpha)}{(2-\alpha)M(\alpha)} A \|c_{n-1}(x,t) - c_{n-2}(x,t)\| \\ &\quad + \tfrac{2\alpha}{(2-\alpha)M(\alpha)} B \int_0^t \{\|c_{n-1}(x,s) - c_{n-2}(x,s)\|\}\, ds \end{aligned} \tag{29}$$

or

$$\begin{aligned} &\|U_n(x,t)\| \\ &\leq \tfrac{2(1-\alpha)}{(2-\alpha)M(\alpha)} A \|U_{n-1}(x,t)\| \\ &+ \tfrac{2\alpha}{(2-\alpha)M(\alpha)} B \int_0^t \{\|U_{n-1}(x,s)\|\}\, ds \end{aligned} \tag{30}$$

□

Theorem 2.2. *Show that the **Oxygen Diffusion Problem Fractional Module** is the model of the oxygen absorption in human body having a coupled-solution.*

Proof. As we have seen that, the above equation (**??**), is bounded, as well as, we have proved that the kernel satisfy the Lipschiz condition, therefore the following results obtained in equation (**??**) using the recursive technique, we get the following relation

$$\begin{aligned} &\|U_n(x,t)\| \\ &\leq \|U(x,0)\| \left\{ \left(\tfrac{2(1-\alpha)}{(2-\alpha)M(\alpha)} A \right)^n + \left(\tfrac{2\alpha}{(2-\alpha)M(\alpha)} Bt \right)^n \right\} \end{aligned} \tag{31}$$

Therefore the above solutions exist and are continuous. Nonetheless, to show that the above is a solution of eqn. (14), we get

$$c(x,t) = c_n(x,t) - P_n(x,t) \tag{32}$$

thus

$$\begin{aligned} G(t) - G_n(t) &= \tfrac{2(1-\alpha)}{(2-\alpha)M(\alpha)} K_1(x,t,c-P_n(t)) \\ &+ \tfrac{2\alpha}{(2-\alpha)M(\alpha)} \int_0^t K_1(s,t,c-P_n(t))\, ds, \end{aligned} \tag{33}$$

It follows from the above that:

$$\begin{aligned} &c(x,t) - \tfrac{2(1-\alpha)}{(2-\alpha)M(\alpha)} K_1(x,t,c) - c(x,0) - \tfrac{2\alpha}{(2-\alpha)M(\alpha)} \int_0^t K_1(s,t,c)\, ds \\ &= P_n(x,t) + \tfrac{2(1-\alpha)}{(2-\alpha)M(\alpha)} K_1(x,t,c) + \tfrac{2\alpha}{(2-\alpha)M(\alpha)} \int_0^t \{K_1(s,t,c-P_n(s,t)) - K_1(s,t,c)\}\, ds. \end{aligned} \tag{34}$$

Now apply the norm on both sides and using the Lipchitz condition, we get

$$\begin{aligned} &\left\| c(x,t) - \tfrac{2(1-\alpha)}{(2-\alpha)M(\alpha)} K_1(x,t,c) - c(x,0) - \tfrac{2\alpha}{(2-\alpha)M(\alpha)} \int_0^t K_1(s,t,c)\, ds \right\| \\ &\leq \|P_n(x,t)\| + \left\{ \tfrac{2(1-\alpha)}{(2-\alpha)M(\alpha)} A + \tfrac{2\alpha}{(2-\alpha)M(\alpha)} Bt \right\} \|P_n(x,t)\|, \end{aligned} \tag{35}$$

On taking limit $n \to \infty$ of equation (35), we get

$$c(x,t) = \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)} K_1(x,t,c) + c(x,0) + \frac{2\alpha}{(2-\alpha)M(\alpha)} \int_0^t K_1(s,t,c)\, ds, \tag{36}$$

Eqn. (36) is the coupled solution of the eqn. (14), hence we can say that solution exists.

□

3. Uniqueness of the Solution

Now in this part, we want to show that the solution presented in the above section is unique.
To prove this, we consider that we can find another solution for system (14), say $c(x,t)$ then:

$$\begin{aligned} c(x,t)-c_1(x,t) &= \tfrac{2(1-\alpha)}{(2-\alpha)M(\alpha)}\{K_1(x,t,c)-K_1(x,t,c_1)\} \\ &+\tfrac{2\alpha}{(2-\alpha)M(\alpha)}\int_0^t\{K_1(s,t,c)-K_1(s,t,c_1)\}\,ds, \end{aligned} \tag{37}$$

apply the norm on the both sides of equation (37),

$$\begin{aligned} \|c(x,t)-c_1(x,t)\| &\le \tfrac{2(1-\alpha)}{(2-\alpha)M(\alpha)}\{\|K_1(x,t,c)-K_1(x,t,c_1)\|\} \\ &+\tfrac{2\alpha}{(2-\alpha)M(\alpha)}\int_0^t\{\|K_1(s,t,c)-K_1(s,t,c_1)\|\}\,ds. \end{aligned} \tag{38}$$

On using the Lipchitz condition, having the fact in mind that the solution is bounded, we get

$$\|c(x,t)-c_1(x,t)\| < \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}HD+\left\{\frac{2\alpha}{(2-\alpha)M(\alpha)}(J_1Dt)\right\}^n \tag{39}$$

this is true for any n hence

$$c(x,t)=c_1(x,t).$$

Hence it shows the uniqueness of the solution of system (14).

4. Application of Fabrizio derivative to Oxygen Diffusion Problem Fractional

To get the best solution of Oxygen Diffusion Problem Fractional Module we use an iterative technique. The method involves the Laplace transform and it's inverse.
Applying the Laplace transform on both sides of (14),we get

$$\frac{pL(c(x,t))-c(x,0)}{p+\alpha(1-p)} = L(c_{xx}-1), \tag{40}$$

or

$$L(c(x,t)) = \frac{c(x,0)}{p}+\frac{(p+\alpha(1-p))}{p}L\left\{\frac{\partial^2 c}{\partial x^2}-1\right\}, \tag{41}$$

applying the inverse Laplace transform on both sides of (41), we get

$$c(x,t)=c(x,0)+L^{-1}\left[\frac{(p+\alpha(1-p))}{p}L\left\{\frac{\partial^2 c}{\partial x^2}-1\right\}\right], \tag{42}$$

We next obtain the following recursive formula from (42)

$$c_{n+1}(x,t)=c_n(x,0)+L^{-1}\left[\frac{(p+\alpha(1-p))}{p}L\left\{\frac{\partial^2 c_n}{\partial x^2}-1\right\}\right], \tag{43}$$

The coupled solution is thus provided as:

$$c(x,t) = \lim_{n\to\infty} c_n(x,t) \tag{44}$$

we get the required solution.

5. Conclusions

In this paper, our aim is to find the possibility of extending the application of the new fractional derivative of without singular kernel in to other fields of science and technology. We have applied the fractional derivative to the Oxygen Diffusion Problem Fractional Module and use the fixed-point theorem to prove the existence and uniqueness of the coupled-solution. A derivation of the special solution was done via an iterative approach. Through this process we can present the biological behavior of the real life problems.

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Mathematics Department, College of Science, King Saud University, Riyadh 11989, Saudi Arabia.
E-mail address: `alhaghog@gmail.com`

Mathematics Department, College of Science, King Saud University, Riyadh 11989, Saudi Arabia
E-mail address: `obalgahtani@ksu.edu.sa`

Department of Mathematics,Yagyavalkya Institute of Technology, Jaipur - 302022, India
E-mail address: `ravimath@gmail.com`

School of Liberal Studies, Ambedkar University Delhi, Delhi-11006, India
E-mail address: `pranaygoswami83@gmail.com`