UNIVERSITATIS IAGELLONICAE ACTA MATHEMATICA, FASCICULUS XXXIX
2001

NONLOCAL PROBLEM FOR THE HYPERBOLIC SYSTEM OF DIFFERENTIAL EQUATION OF THE FIRST ORDER

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Abstract. In the present paper we consider the nonlocal problem for the system of hyperbolic equations of the first order in two independet variables in the case when nonlinear functions satisfy Caratheodory assumptions. Some conditions for uniqueness and existence of a solution are obtained.

Nonlocal problems for hyperbolic equations of the first order describe the dynamic of population [**1**]–[**3**]. During the last thirty years the existence and uniqueness of the solution of nonlocal problems for the system of hyperbolic equations have been considered in a number of papers [**4**]–[**8**]. The authors assumed that the nonlinear functions satisfy the Lipschitz condition with respect to the unknown functions. In this paper we consider the nonlocal problem for the system of hyperbolic equations of the first order in two independent variables in the case when nonlinear functions satisfy Caratheodory conditions.

We shall consider the system of hyperbolic equation of the form

$$\begin{aligned} &u_t(x,t) + A(x)u_x(x,t) + C(x,t)u(x,t) + \\ &\quad + G(x,u) + \int_a^b Q(\xi,t)u(\xi,t)d\xi = F(x,t) \end{aligned} \tag{1}$$

in the domain

$$\Omega_T = \{(x,t) \,:\, 0 < x < c,\ 0 < t < T\}, \quad T < \infty,$$

For this system we put the following boundary and initial conditions:

$$u(0,t) = \Lambda u(c,t) \tag{2}$$

$$u(x,0) = \Phi(x) \tag{3}$$

where A, C, Q, Λ are square matrices of order n, and

$$u = (u_1, ..., u_n)^T, \quad G = (g_1, ..., g_n)^T,$$

$$F = (f_1, ..., f_n)^T, \quad \Phi = (\phi_1, ..., \phi_n)^T, \quad 0 \le a < b \le c.$$

For equation (1), we consider the following conditions:

(A) $A \in C^1_{n^2}([0, c])$; $\det A(x) \neq 0$, $A(x) = A^t(x)$
for every $\xi \in R^n$ and $x \in [0, c]$;
$A(c) - \Lambda^t A(0)\Lambda = 0$.

(G) The function G is continous with respect to ξ for almost all $x \in (0, c)$ and measurable with respect to x for every $\xi \in R^n$ and satisfies the following inequalities:
$(G(x, \xi) - G(x, \mu), \xi - \mu) \ge G_0|\xi - \mu|^p$,
where $2 < p < \infty$, $G_0 \ge 0$,
$|g_i(x, \xi_1, ..., \xi_n)| \le G_1 \sum\limits_{j=1}^{n} |\xi_j|^{p-1}, \quad G_1 > 0$
for $i = 1, ..., n$ and for every $\xi \in R^n$ and almost all $x \in (0, c)$.
By $(\cdot, \cdot)$ we denote the scalar product in R^n.

Definition 1. We call a function u a solution of problem (1)–(3) if

$$u \in W^{1,2}_n((0, T); L^2(0, c)), \quad u_x \in L^2_n(\Omega_T) + L^q_n(\Omega_T), \quad \frac{1}{p} + \frac{1}{q} = 1$$

and u satisfies (1), (2), (3) for almost all $(x, t) \in \Omega_T$.

Denote

$$Q_0 := \sup_{a<x<b, 0<t<T} \|Q(x, t)\|$$

where $\| \cdot \|$ is the Euclidean norm of the matrix Q.

Theorem 1. *If the conditions* (A), (G) *hold and* $C, Q \in L^\infty_{n^2}(\Omega_T)$ *then the problem* (1)–(3) *has at most one solution.*

Proof. To obtain a contradiction, suppose that there exist two solutions u^1, u^2 of the problem (1)–(3) such that $u^1 \neq u^2$. Denote $u = u^1 - u^2$. It is easy to show that for every $\tau \in (0, T]$ the following equality is satisfied

$$\begin{aligned} \int\limits_{\Omega_\tau} \Big[& (u_t(x,t), u(x,t)) + (A(x)u_x(x,t), u(x,t)) + \\ & + (C(x,t)u(x,t), u(x,t)) + ((G(x,u^1) - G(x,u^2), u(x,t)) + \\ & + \int\limits_a^b (Q(\xi,t)u(\xi,t), u(x,t))d\xi \Big] e^{-\lambda t} dxdt = 0 \end{aligned} \tag{4}$$

where $\lambda > 0$ and $u(x,0) = 0$. Hence if we consider the respective components of the last equality we will have

$$I_1 = \int\limits_{\Omega_\tau} (u_t(x,t), u(x,t)) e^{-\lambda t} dxdt =$$
$$\frac{\lambda}{2} \int\limits_{\Omega_\tau} |u(x,t)|^2 e^{-\lambda t} dxdt + \frac{1}{2} \int\limits_0^c |u(x,\tau)|^2 e^{-\lambda \tau} dx.$$

and

$$I_2 = \int\limits_{\Omega_\tau} (A(x) u_x(x,t), u(x,t)) e^{-\lambda t} dxdt =$$
$$= \frac{1}{2} \int\limits_{\Omega_\tau} (A(x) u(x,t), u(x,t))_x e^{-\lambda t} dxdt$$
$$- \frac{1}{2} \int\limits_{\Omega_\tau} (A_x(x) u(x,t), u(x,t)) e^{-\lambda t} dxdt.$$

From (A) we have

$$I_2 \geq -\frac{1}{2} A_1 \int\limits_{\Omega_\tau} |u(x,t)|^2 e^{-\lambda t} dxdt,$$

where $A_1 = \sup\limits_{[0,c]} \|A_x(x)\|$. Since $C \in L^\infty_{n^2}(\Omega_T)$, we obtain

$$I_3 = \int\limits_{\Omega_\tau} (C(x,t) u(x,t), u(x,t)) e^{-\lambda t} dxdt \geq c_0 \int\limits_{\Omega_\tau} |u(x,t)|^2 e^{-\lambda t} dxdt.$$

By (G)

$$I_4 = \int\limits_{\Omega_\tau} (G(x,u^1) - G(x,u^2), u(x,t)) e^{-\lambda t} dxdt \geq 0,$$

and since $Q \in L^\infty_{n^2}(\Omega_T)$, we have

$$I_5 = \int\limits_{\Omega_\tau} \int\limits_a^b (Q(\xi,t) u(\xi,t), u(x,t)) d\xi e^{-\lambda t} dxdt \leq$$
$$\leq \int\limits_{\Omega_\tau} \int\limits_a^b \|Q(\xi,t)\| |u(\xi,t)| d\xi |u(x,t)| e^{-\lambda t} dxdt \leq$$
$$\leq \frac{1}{2} Q_0 (c(b-a)+1) \int\limits_{\Omega_\tau} |u(x,t)|^2 e^{-\lambda t} dxdt.$$

Thus we get the following inequality

$$\int_0^c |u(x,\tau)|^2 e^{-\lambda\tau} dx + \\ + (\lambda + 2c_0 - A_1 - Q_0(c(b-a)+1)) \int_{\Omega_\tau} |u(x,t)|^2 e^{-\lambda t} dx dt \leq 0 \tag{5}$$

for $\tau \in (0,T]$. We choose λ such that

$$\lambda + 2c_0 - A_1 - Q_0(c(b-a)+1) \geq 0.$$

Then

$$\int_0^c |u(x,t)|^2 \leq 0, \quad t \in (0,T),$$

which means that $u(x,t) = 0$ for almost all $(x,t) \in \Omega_T$.

This completes the proof of Theorem 1. □

Denote by J the Jacobian matrix of the function $G(x,u)$

$$J = \left[\frac{\partial g_i(x,u)}{\partial u_j}\right]_{i,j=1}^n.$$

Let $\widehat{W}_n^{1,2}$ be the closure of the function space $C_n^1([0,c])$, satisfying (2) with respect to the norm of the space $W_n^{1,2}(0,c)$.

Theorem 2. *Suppose that the conditions* (A) *and* (G) *hold and* $C, C_t, Q, Q_t \in L_{n^2}^\infty(\Omega_T)$; $F, F_t \in L_n^2(\Omega_T)$; $\Phi \in \widehat{W}_n^{1,2}(0,c)$. *Moreover, assume that*

$$(J(x,\mu)\xi, \xi) \geq 0 \tag{6}$$

for every $\mu, \xi \in R^n$ *and almost every* $x \in (0,c)$. *Then there exists a solution of the problem* (1)–(3).

Proof. Consider the following problem for eigenfunctions:

$$y'' = \lambda y, \tag{7}$$

$$y(0) = \Lambda y(c); \; y'(c) = \Lambda^T y(0) \tag{8}$$

where $y = (y_1, ..., y_n)^T$. Then there exists an orthogonal system of eigenfunctions $\{w^k(x)\}$, $w^k(x) = (w_1^k(x), ..., w_n^k(x))^T$, of the problem (7), (8), which is a basis of the space $L_n^2(0,c)$. We consider a sequence of functions of the form

$$u^N(x,t) = \sum_{k=1}^N C_k^N(t) w^k(x)$$

for $N = 1, 2, ...$ where the functions $C_1^N, ..., C_N^N$ constitute the solution of the following Cauchy problem:

$$
\begin{aligned}
&\int_0^c \Big[(u_t^N(x,t), w^k(x)) + (A(x)u_x^N(x,t), w^k(x)) + \\
&\quad + (C(x,t)u^N(x,t), w^k(x)) + (G(x,u^N), w^k(x)) \\
(9) \qquad &\quad + \int_a^b (Q(\xi,t)u^N(\xi,t), w^k(x))d\xi - \\
&\quad - (F(x,t), w^k(x)) \Big] dx = 0 \quad for \quad k = 1, ..., N,
\end{aligned}
$$

with

$$
(10) \qquad C_k^N(0) = \phi_k^N \quad for \quad k = 1, ..., N,
$$

where

$$
\Phi^N(x) = \sum_{k=1}^N \phi_k^N w^k(x)
$$

and

$$
\|\Phi^N - \Phi\|_{\widehat{W}_n^{2,1}(0,c)} \to 0 \quad if \quad N \to \infty.
$$

Observe that the assumptions of Theorem 2 guarantee the existence of the solution of the problem (9), (10), which is differentiable in the interval $(0, T)$. Multiplying (9) by the functions $C_k^N(t)e^{-\lambda t}$, respectively, then summing by k from 1 to N and integrating with respect to t from 0 to τ, $\tau \in (0, T]$ we obtain

$$
\begin{aligned}
&\int_{\Omega_\tau} \Big[(u_t^N(x,t), u^N(x,t)) + (A(x)u_x^N(x,t), u^N(x,t)) + \\
&\quad + (C(x,t)u^N(x,t), u^N(x,t)) + (G(x,u^N), u^N(x,t)) + \\
(11) \qquad &\quad + \int_a^b (Q(\xi,t)u^N(\xi,t), u^N(x,t))d\xi \\
&\quad - (F(x,t), u^N(x,t)) \Big] e^{-\lambda t} dxdt = 0.
\end{aligned}
$$

As in the proof of Theorem 1 we obtain

$$I_6=\int\limits_{\Omega_\tau}\Big[(u_t^N(x,t),u^N(x,t))+(A(x)u_x^N(x,t),u^N(x,t))+$$

$$+(C(x,t)u^N(x,t),u^N(x,t))+\int\limits_a^b(Q(\xi,t)u^N(\xi,t),u^N(x,t))d\xi\Big]e^{-\lambda t}dx\geq$$

$$\geq\frac{1}{2}\int\limits_0^c|u^N(x,\tau)|^2e^{-\lambda\tau}dx-\frac{1}{2}\int\limits_0^c|\Phi^N(x)|^2dx+$$

$$+\frac{1}{2}(\lambda+2c_0-A_1-Q_0(c(b-a)+1))\int\limits_{\Omega_\tau}|u^N(x,t)|^2e^{-\lambda t}dxdt.$$

Moreover, from (G) we have

$$I_7=\int\limits_{\Omega_\tau}(G(x,u^N),u^N(x,t))e^{-\lambda t}dxdt\geq$$

$$\geq G_0\int\limits_{\Omega_\tau}(|u^N(x,t)|^pe^{-\lambda t}dxdt$$

and

$$I_8=\int\limits_{\Omega_\tau}(F(x,t),u^N(x,t))e^{-\lambda t}dxdt\leq$$

$$\frac{1}{2}\int\limits_{\Omega_\tau}|F(x,t)|^2e^{-\lambda t}dxdt+\frac{1}{2}\int\limits_{\Omega_\tau}|u^N(x,t)|^2e^{-\lambda t}dxdt.$$

If we choose now

$$\lambda=\max\{A_1+Q_0(c(b-a)+1)+1-2c_0,3\}$$

then from the estimates of I_6,I_7,I_8 and (11), for N large enough, we obtain the following inequality

$$2G_0\int\limits_{\Omega_\tau}|u^N(x,t)|^pe^{\lambda t}dxdt+\int\limits_0^c|u^N(x,\tau)|^2dxe^{\lambda\tau}\leq$$

(12)

$$\leq e^{\lambda\tau}\Big(2\int\limits_0^c|\Phi(x)|^2dx+\int\limits_{\Omega_\tau}|F(x,t)|^2dxdt\Big)$$

where $\tau\in[0,T]$. Differentiating (9) with respect to t, then multiplying by functions $C_{kt}^N(t)e^{-\lambda t}$, respectively, summing by k from 1 to N and integrating by t from 0 to τ we obtain

$$
\begin{aligned}
&\int\limits_{\Omega_\tau}\Big[(u_{tt}^N(x,t),u_t^N(x,t))+(A(x)u_{xt}^N(x,t),u_t^N(x,t))+\\
&+(C(x,t)u_t^N(x,t),u_t^N(x,t))+\int\limits_a^b(Q(\xi,t)u_t^N(\xi,t),u_t^N(\xi,t))d\xi-\\
&-(F(x,t),u_t^N(x,t))+(C_t(x,t)u^N(x,t),u_t^N(x,t))+\\
&+(J(x,u^N)u_t^N(x,t),u_t^N(x,t))\\
&+\int\limits_a^b(Q_t(\xi,t)u^N(\xi,t),u_t^N(x,t))d\xi\Big]e^{-\lambda t}dxdt=0.
\end{aligned} \tag{13}
$$

Again, it is easy to estimate

$$
\begin{aligned}
I_9 &= \int\limits_{\Omega_\tau}\Big[(u_{tt}^N(x,t),u_t^N(x,t))+(A(x)u_{xt}^N(x,t),u_t^N(x,t))+\\
&+(C(x,t)u_t^N(x,t),u_t^N(x,t))+\int\limits_a^b(Q(\xi,t)u_t^N(\xi,t),u_t^N(x,t))d\xi-\\
&-(F_t(x,t),u_t^N(x,t))\Big]e^{-\lambda t}dxdt\geq\\
&\geq\frac{1}{2}\int\limits_0^c|u_t^N(x,\tau)|^2e^{-\lambda\tau}dx-\frac{1}{2}\int\limits_0^c|u_t^N(x,0)|^2dx-\\
&-\frac{1}{2}\int\limits_{\Omega_\tau}|F_t(x,t)|^2e^{-\lambda t}dxdt+\frac{1}{2}(\lambda+2c_0-A_1-1-\\
&-Q_0(c(b-a)+1))\int\limits_{\Omega_\tau}|u_t^N(x,t)|^2e^{-\lambda t}dxdt.
\end{aligned}
$$

Next, from the assumptions of Theorem 2 we have

$$
\begin{aligned}
I_{10}&=\int\limits_{\Omega_\tau}(C_t(x,t)u^N(x,t),u_t^N(x,t))e^{-\lambda t}dxdt\leq\\
&\leq\frac{1}{2}\int\limits_{\Omega_\tau}(|u_t^N(x,t)|^2+\sup_{Q_T}\|C_t(x,t)\|^2|u^N(x,t)|^2)e^{-\lambda t}dxdt,
\end{aligned}
$$

$$I_{11} = \int\limits_{\Omega_\tau} (J(x, u^N) u_t^N(x,t), u_t^N(x,t)) dx dt \geq 0$$

and

$$I_{12} = \int\limits_{\Omega_\tau} \int\limits_a^b (Q_t(\xi,t) u^N(\xi,t), u_t^N(x,t)) d\xi dx dt \leq \frac{1}{2} \int\limits_{\Omega_\tau} [|u_t^N(x,t)|^2 + \\ + \sup_{a<x<b, 0<t<T} \|Q_t(x,t)\|^2 (c(b-a)+1) |u^N(x,t)|^2 e^{-\lambda t} dx dt.$$

To estimate the integral

$$\int\limits_0^c |u_t^N|^2 dx$$

we again use (9). Hence we obtain

$$\begin{aligned} \int\limits_0^c \Big[& |u_t^N(x,0)|^2 + (A(x) u_x^N(x,0), u_t^N(x,0)) + \\ & + (G(x,u^N), u_t^N(x,0)) + \int\limits_a^b (Q(\xi,0) u^N(\xi,0), u_t^N(x,0)) d\xi + \\ & + (C(x,0) u^N(x,0), u_t^N(x,0)) - (F(x,0), u_t^N(x,0)) \Big] dx = 0. \end{aligned} \tag{14}$$

Thus

$$\begin{aligned} I_{13} = \int\limits_0^c \Big[& (A(x) u_x^N(x,0), u_t^N(x,0)) + (C(x,0) u^N(x,0), u_t^N(x,0)) + \\ & + (G(x,u^N), u_t^N(x,0)) + \int\limits_a^b (Q(\xi,0) u^N(\xi,0), u_t^N(x,0)) d\xi - \\ & - (F(x,0), u_t^N(x,0)) \Big] dx \leq \frac{1}{2} \int\limits_0^c |u_t^N(x,0)|^2 dx + \\ & + \frac{\mu_1}{2} \int\limits_0^c (|\Phi(x)|^2 + |\Phi_x(x)|^2) dx, \end{aligned}$$

where the constant μ_1 depends on matrices A, C, Q, the function $F(x,0)$ and constants G_1, n, p. From (14) we obtain the following estimation

$$\int_0^c |u_t^N(x,0)|^2 dx \leq \mu_1 \int_0^c (|\Phi(x)|^2 + |\Phi_x(x)|^2) dx. \tag{15}$$

From the estimates of the integrals $I_9, I_{10}, I_{11}, I_{12}$ and from (12), (15) we obtain the inequality

$$\begin{aligned} \int_0^c |u_t^N(x,\tau)|^2 dx \leq \mu_2 \Big[\int_0^c (|\Phi(x)|^2 + |\Phi_x(x)|^2) dx + \\ + \int_{\Omega_\tau} (|F(x,t)|^2 + |F_t(x,t)|^2) dx dt \Big] \end{aligned} \tag{16}$$

for $\tau \in (0,T]$, where the constanst μ_2 does not depend on N. Moreover, from the assumptions (G) and (12)

$$\begin{aligned} \int_{\Omega_\tau} |g_i(x,u^N)|^q dx dt \leq \int_{\Omega_\tau} (G_1 \sum_{i=1}^n |u_i^N|^{p-1})^q dx dt \leq \\ \leq \mu_3 \int_{\Omega_\tau} (|u^N(x,t)|^p dx dt \leq \mu_4 \end{aligned} \tag{17}$$

for $\tau \in (0,T]$, $i = 1, ..., n$. By inequalities (12), (15), (17) there exists a subsequence $\{u^m(x,t)\}$ of the sequence $\{u^N(x,t)\}$ such that

$$u^m \to u \quad \text{weakly in} \quad L_n^2(\Omega_T)$$

$$u_t^m \to u_t \quad \text{weakly in} \quad L_n^2(\Omega_T)$$

$$G(x,u^m) \to \omega \quad \text{weakly in} \quad L_n^q(\Omega_T)$$

when $m \to \infty$.

Now we consider a sequence $\{y_m\}$ defined by the formula

$$
\begin{aligned}
0 \le y_m \quad = \quad & \int\limits_{\Omega_T} e^{-\lambda t}(G(x,u^m) - G(x,v), u^m(x,t) - v(x,t))dxdt + \\
& + \int\limits_{\Omega_T} e^{-\lambda t}(G(x,v), u^m(x,t) - v(x,t))dxdt - \\
& - \int\limits_{\Omega_T} e^{-\lambda t}(G(x,u^m), v(x,t))dxdt + \\
& + \int\limits_{\Omega_T} e^{-\lambda t}(G(x,u^m), u^m(x,t))dxdt = \int\limits_{\Omega_\tau} e^{-\lambda t}\Big[(F(x,t), u^m(x,t)) + \\
& + \frac{1}{2}(A_x(x)u^m(x,t), u^m(x,t)) - (C(x,t)u^m(x,t), u^m(x,t)) - \\
& - (u_t^m(x,t), u^m(x,t)) - \int\limits_a^b (Q(\xi,t)u^m(\xi,t), u^m(x,t))d\xi\Big]dxdt - \\
& - \int\limits_{\Omega_T} e^{-\lambda t}[(G(x,v), u^m(x,t) - v(x,t)) + (G(x,u^m), v(x,t))]dxdt,
\end{aligned}
$$

where v is an arbitrary function in $L_n^p(\Omega_T)$. It is easy to prove that for the same λ the following inequality holds

$$
\begin{aligned}
0 \le y_m \le & \int\limits_{\Omega_T} e^{-\lambda t}\Big[(F(x,t), u(x,t)) + \frac{1}{2}(A_x(x)u(x,t), u(x,t)) - \\
& - \frac{\lambda}{2}(u(x,t), u(x,t)) - (C(x,t)u(x,t), u(x,t)) - \\
& \int\limits_a^b (Q(\xi,t)u(\xi,t), u(x,t))d\xi\Big]dxdt + \frac{1}{2}\int\limits_0^c |\Phi(x)|^2 dx - \\
& - \int\limits_{\Omega_\tau} e^{-\lambda t}[(\omega, v) + (G(x,v), u(x,t) - v(x,t))]dxdt \\
& - \frac{1}{2}\int\limits_0^c e^{-\lambda\tau}|u(x,\tau)|^2 dx.
\end{aligned}
\tag{18}
$$

On the other hand, from (9) we obtain that for every $v \in \widehat{W}_n^{2,1}(\Omega_T)$ the following equality holds

$$\begin{aligned} &\int_{\Omega_T} e^{-\lambda t}\Big[(u_t(x,t), u(x,t)) - (A_x(x)u(x,t), v(x,t)) - \\ (19)\quad &- (A(x)u(x,t), v_x(x,t)) + (C(x,t)u(x,t), v(x,t)) - \\ &- \int_a^b (Q(\xi,t)u(\xi,t), v(x,t))d\xi + (\omega, v(x,t)) - (F(x,t), v(x,t))\Big] dxdt = 0. \end{aligned}$$

Using (A) and (19) we find that $u_x \in L_n^2(\Omega_T) + L_n^q(\Omega_T)$. Thus $u \in L_n^\infty(\Omega_T)$. Then if we put the function u instead of the function v in (19) we obtain

$$\begin{aligned} &\int_{\Omega_T} e^{-\lambda t}\Big[\frac{\lambda}{2}(u(x,t), u(x,t)) - \frac{1}{2}(A_x(x)u(x,t), u(x,t)) + \\ &+ (C(x,t)u(x,t), u(x,t)) - \int_a^b (Q(\xi,t)u(x,t), u(x,t))d\xi \\ (20)\quad &+ (\omega, u(x,t)) - (F(x,t), u(x,t))\Big] dxdt + \\ &+ \frac{1}{2}\int_0^c e^{-\lambda T}|u(x,T)|^2 dx - \frac{1}{2}\int_0^c |\Phi(x)|^2 dx = 0. \end{aligned}$$

Adding (19) and (20) we get

$$\int_{\Omega_T} (\omega - G(x,v), u(x,t) - v(x,t)) e^{-\lambda t} dxdt \geq 0. \tag{21}$$

Let $v = u - \alpha w$, $\alpha > 0$, $w \in \widehat{W}_n^{2,1}(\Omega_T)$. Then

$$\int_{\Omega_T} (\omega - G(x,u), w) e^{-\lambda t} dxdt = 0$$

for every $w \in L_n^p(\Omega_T)$, which means that

$$\omega = G(x,u).$$

From (20) we obtain that u is the solution of the problem (1)–(3), which completes the proof of Theorem 2. □

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Received March 6, 2000

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