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TRANSVERSAL DOMINATION IN DOUBLE GRAPHS

S. R. Nayaka[1], Puttaswamy[1] and K. N. Prakash[2]

[1] P.E.S. College of Engineering, Mandya, Karnataka 571401, India;
[2] Vidyavardhaka College of Engineering,
P.B. No.206, Gokulam III Stage, Mysuru 570002, Karnataka, India
E-mail: nayaka.abhi11@gmail.com, prof.puttaswamy@gmail.com,
prakashamaths@gmail.com

Abstract. Let G be any graph. A subset S of vertices in G is called a dominating set if each vertex not in S is adjacent to at least one vertex in S. A dominating set S is called a transversal dominating set if S has nonempty intersection with every dominating set of minimum cardinality in G. The minimum cardinality of a transversal dominating set is called the transversal domination number denoted by $\gamma_{td}(G)$. In this paper, we are considering special types of graphs called double graphs obtained through a graph operation. We study the new domination parameter for these graphs. We calculate the exact value of domination and transversal domination number in double graphs of some standard class of graphs. Further, we also estimate some simple bounds for these parameters in terms of order of a graph.

Key words: transversal dominating set, transversal domination number, direct product, double graph.

Mathematical Subject Classification (2010): 05C69.

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1. Introduction

Let G be a graph. A subset S of vertices is called a *dominating set* of G if every vertex not in S is adjacent to at least one vertex in S. The minimum cardinality of a dominating set is called the *domination number*, denoted by $\gamma(G)$. For any graph G, there may be many dominating sets of different cardinalities between $\gamma(G)$ and the order of G. The concept of transversal domination in graphs is defined and studied in [1]. A dominating set is called the *transversal dominating set* if it intersects every minimum dominating set in G. The minimum cardinality of a transversal dominating set is called the *transversal domination number*, denoted by $\gamma_{td}(G)$. In [1], authors have obtained fundamental results related to transversal domination parameter including exact values for standard graphs and bounds in terms of order and domination number.

Let G and H be any two graphs. The direct product of G and H is a graph denoted by $G \times H$ with the vertex set $V(G) \times V(H)$ such that two vertices (v_1, w_1) and (v_2, w_2) are adjacent in $G \times H$ if and only if v_1 and v_2 are adjacent in G and w_1 and w_2 are adjacent in H. The *total graph* T_n of order n is the graph associated to the total relation (where every vertex is adjacent to each vertex). In fact, T_n can be obtained from the complete graph K_n by adding a loop to every vertex. Given a simple graph G, the double of G is a simple graph denoted by $\mathfrak{D}(G)$ and is defined by $\mathfrak{D}(G) = G \times T_2$. In the double graph $\mathfrak{D}(G)$, two vertices (v_1, w_1) and (v_2, w_2) are adjacent if and only if v_1 and v_2 are adjacent in G.

From the definition of a double graph [2], it follows that if G is a graph of order n and size m then $\mathfrak{D}(G)$ is a graph of order $2n$ and size $4m$. In particular, the degree of a vertex (v, k) will be $2\deg_G v$. The pentagonal prism with modified lateral edges and its double graph are as shown in Figure 1. The double graph $\mathfrak{D}(G)$ always decomposes into two subgraphs G_0 and G_1 such that $G_0 \cap G_1 = \varnothing$ and $G_0 \cup G_1$ is a spanning subgraph of $\mathfrak{D}(G)$. Then $\{G_0, G_1\}$ is called the decomposition of $\mathfrak{D}(G)$. The double graph operation is defined for any graph G, throughout this paper, by a graph G, we mean a graph without loops and multiple edges. The multi-star graph $K_m(a_1, a_2, \ldots, a_m)$ is a graph of order $a_1 + a_2 + \cdots + a_m + m$ formed by joining $a_1, a_2, \ldots, a_m$ end-edges to m vertices of K_m. For example, $K_2(a_1, a_2)$ is a double star.

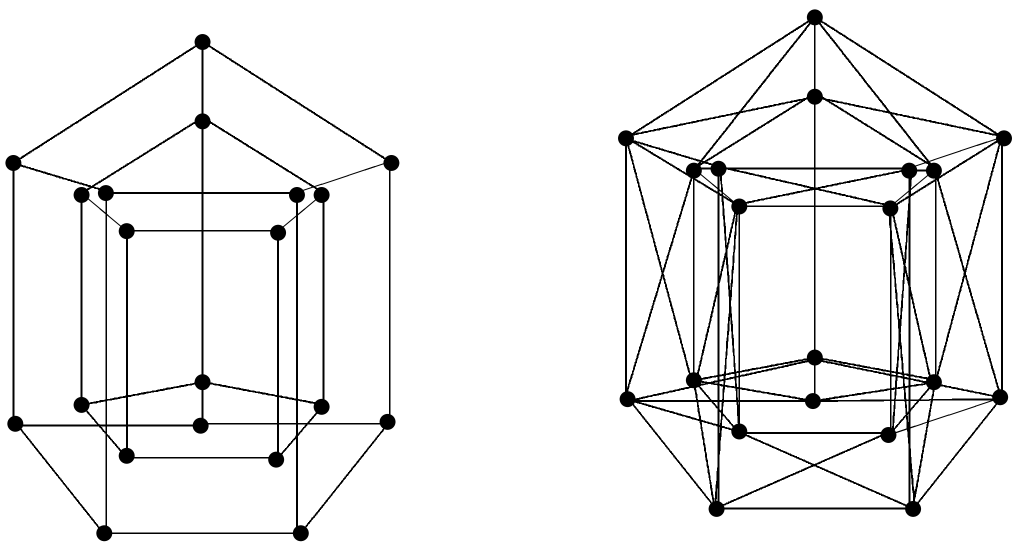

Fig. 1. Double graph of a Pentagonal prism.

Lemma 1.1. *Let G be a path of order n. Then*

$$\gamma(\mathfrak{D}(G)) = \begin{cases} 2\lfloor \frac{n}{3} \rfloor, & \text{if } n \equiv 0 \text{ or } 1\,(mod\ 3); \\ 2\lfloor \frac{n}{3} \rfloor + 1, & \text{if } n \equiv 2\,(mod\ 3). \end{cases}$$

$\lhd$ Let G be a path of order n. Then $\mathfrak{D}(G)$ is a $\{2, 4\}$-regular graph of order $2n$. First, suppose $n \equiv 0$ or $1\,(\mathrm{mod}\ 3)$. Let S' be a minimum dominating set in G. For each vertex u'_i of S', attach a vertex v_{i+1} from another copy of G, which is adjacent to u' in $\mathfrak{D}(G)$. The resulting set S of cardinality $2\gamma(G)$ dominates $\mathfrak{D}(G)$ and minimality holds since each vertex in S has a private neighbor. Thus, $\gamma(\mathfrak{D}(G)) = 2\lfloor \frac{n}{3} \rfloor$. Finally, assume $n \equiv 2(\mathrm{mod}\ 3)$. Let v be a pendant vertex of G and let G' be a graph obtained by removing the vertex v. Then, clearly, $\gamma(\mathfrak{D}(G)) = \gamma(\mathfrak{D}(G')) + 1$. Since, G' will be isomorphic to a path of order $3n$ or $3n+1$, it follows that $\gamma(\mathfrak{D}(G)) = 2\lfloor \frac{n}{3} \rfloor + 1$. $\rhd$

Theorem 1.1. *Let G be a path of order $n \geqslant 3$. Then $\gamma_{td}(\mathfrak{D}(G)) = \gamma(\mathfrak{D}(G)) + 1$.*

$\lhd$ Let G be a path of order n. Since the γ-set of $\mathfrak{D}(G)$ is obtained by choosing vertices from the γ-set of copies of G, it will be clear that there are atmost two possibilities to select vertices from a γ-set of copies of G. Thus, for any vertex u of γ-set of G which is not in γ-set

S of $\mathfrak{D}(G)$, the set $S \cup \{u\}$ will be a dominating set intersecting the minimum dominating sets in $\mathfrak{D}(G)$. Minimality of the set $S_1 = S \cup \{u\}$ since for any vertex v of S_1, there always exists a γ-set of $\mathfrak{D}(G)$ not intersecting S_1. Hence, $\gamma_{td}(\mathfrak{D}(G)) = \gamma(\mathfrak{D}(G)) + 1$. $\rhd$

Lemma 1.2. *Let G be a complete graph. Then $\gamma(\mathfrak{D}(G)) = 2$.*

Theorem 1.2. *Let G be a complete graph of order n. Then $\gamma_{td}(\mathfrak{D}(G)) = 2n - 1$.*

$\lhd$ Let G be a complete graph of order n. Then $\mathfrak{D}(G)$ will be a regular graph order $(2n-2)$. Since every pair of vertices, taken from each copies of G, is a dominating set it follows that $\gamma_{td}(\mathfrak{D}(G)) = 2n - 1$. $\rhd$

Theorem 1.3. *Let G be a cycle of order $n \geqslant 4$. Then*

$$\gamma(\mathfrak{D}(G)) = \begin{cases} 5, & \textit{if } n = 4; \\ 3, & \textit{if } n = 5; \\ 2\lfloor \frac{n}{3} \rfloor, & \textit{otherwise.} \end{cases}$$

$\lhd$ Let G be a cycle of order $n \geqslant 3$. Then $\mathfrak{D}(G)$ is a 4-regular graph of order $2n$. If $n = 4$, then clearly $\gamma(\mathfrak{D}(G)) = 5$. Assume $n = 5$. Then, any minimum dominating set of a copy of G, in which a vertex u of S replaced by the corresponding vertex u', will be a minimum dominating set and so $\gamma(\mathfrak{D}(G)) = 3$.

Now, suppose $n \geqslant 6$. We may consider three possible cases here. First, suppose $n = 3k$. As the graph $\mathfrak{D}(G)$ consists of two copies of C_n, choose a minimum dominating set S' of one copy, which dominates $\mathfrak{D}(G)$ except the vertices corresponding to the vertices of S'. So that, the set S obtained by taking the vertices not dominated by S' together with S', will be a dominating set of $\mathfrak{D}(G)$. Further, for any vertex v of S, the set $S - \{v\}$ will not be a dominating set in G and so in $\mathfrak{D}(G)$. Therefore, $\gamma(\mathfrak{D}(G)) = 2|S'| = 2\lfloor \frac{n}{3} \rfloor$.

Next, suppose $n = 3k + 1$. As in the previous case, choose a minimum dominating set S' of a copy G and then select the corresponging vertices from another copy of G. This will be a dominating set but not minimal as the vertices v_1 and v'_n have two neighbors in the set. Hence, $S' - \{v_1, v'_n\}$ will be a minimum dominating set in G. Therefore, $\gamma(\mathfrak{D}(G)) = |S' - \{v_1, v'_n\}| = 2\lfloor \frac{n}{3} \rfloor$. Finally, if $n = 3k+2$, similar to the above case, for any set S' consists of vertices from γ-set of G and the corresponding vertices in other copy of G, the set $S = (S' - \{v'_1, v_{n-1}\}) \cup \{v'_n\}$ will be a minimum dominating set of cardinality $2\lfloor \frac{n}{3} \rfloor$. $\rhd$

Theorem 1.4. *Let G be a cycle of order $n \geqslant 3$. Then*

$$\gamma_{td}(\mathfrak{D}(G)) = \begin{cases} 2\lfloor \frac{n}{3} \rfloor + 3, & \textit{if } n \equiv 0 \textit{ or } 1 \,(\textit{mod } 3); \\ \frac{2(n+4)}{3}, & \textit{if } n \equiv 2 \,(\textit{mod } 3). \end{cases}$$

$\lhd$ Let G be a cycle of order $n \geqslant 3$ and let $V(\mathfrak{D}(G)) = \{v_i, v'_j : 1 \leqslant i, j \leqslant n\}$. First we note that any minimum dominating set in $\mathfrak{D}(G)$ contains a vertex from $V' = \{v_1, v_2, v'_2, v_n, v_n, \}$. Let H be a spanning sub-graph of G having the vertex set $V - V'$. Then $\gamma_{td}(\mathfrak{D}(G)) = \gamma(H) + |V'|$. Further, it can be noted that H will be isomorphic to a double graph $\mathfrak{D}(P_{n-3})$. Therefore, $\gamma_{td}(\mathfrak{D}(G)) = \gamma(\mathfrak{D}(P_{n-3})) + 5$, establishing the result. $\rhd$

Theorem 1.5. *Let $G \cong K_m(a_1, a_2, \ldots, a_m)$ be a multi-star. Then $\gamma_{td}(\mathfrak{D}(G)) = m + 1$.*

$\lhd$ Let $G \cong K_m(a_1, a_2, \ldots, a_m)$ be a multi-star of order $a_1+a_2+\cdots+a_m$. Clearly $\gamma(G) = m$. Consider the double graph of G and the minimum dominating set S. As every vertex in S covers the leaves adjacent to it and the vertices adjacent to the corresponding vertices in another copy, it follows that S itself a minimum dominating set in $\mathfrak{D}(G)$. Therefore, $\gamma(\mathfrak{D}(G)) = |S| = m$. Finally, since $\mathfrak{D}(G)$ contains exactly two vertex disjoint dominating sets, $\gamma_{td}(\mathfrak{D}(G)) = m + 1$. $\rhd$

DEFINITION 1.1. For $m \geqslant 2$, Jahangir graph $J_{n,m}$ is a graph of order $nm+1$, consisting of a cycle of order nm with one vertex adjacent to exactly m vertices of C_{nm} at a distance n to each other. Jahangir graph $J_{2,16}$ is shown in figure 1.

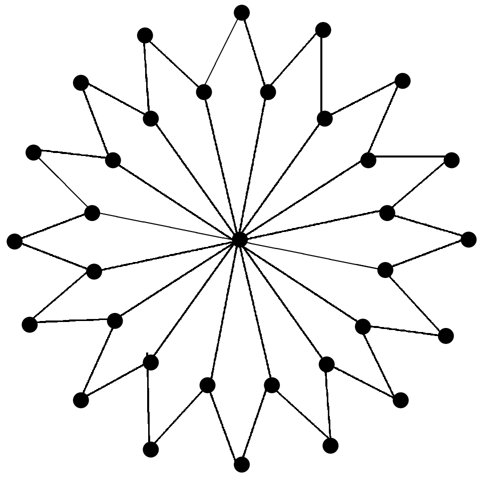

Fig. 2. $J_{2,16}$.

Proposition 1.1 [3]. *Let $G \cong J_{2,m}$ be a Jahangir graph with $m \geqslant 3$. Then*

$$\gamma(G) = \begin{cases} 2, & \text{if } m = 3; \\ \lceil \frac{m}{2} \rceil + 1, & \text{otherwise.} \end{cases}$$

Theorem 1.6. *Let $G \cong J_{n,m}$ be a Jahangir graph with $m, n \geqslant 3$. Then*

$$\gamma(G) = \begin{cases} \frac{m(n-1)}{3} + 1, & \text{if } n \equiv 1\,(mod\ 3); \\ \lceil \frac{mn}{3} \rceil, & \text{if } n \equiv 0 \text{ or } 2\,(mod\ 3). \end{cases}$$

$\lhd$ Let $G \cong J_{n,m}$ be a Jahangir graph with $m, n \geqslant 3$ and let $V(G) = \{v_1, v_2, \ldots, v_{nm}, v_{nm+1}\}$, where v_{nm+1} is the vertex at the center, adjacent to vertices of C_{nm}. First assume $n \equiv 1(\text{mod } 3)$, i. e., $n = 3k+1$, for some positive integer k. From the definition, the vertex v_{nm+1} is adjacent to m vertices of C_{nm} at a distance $3k+1$. Removing the vertex v_{nm+1} from G, the graph induced by $V(G) - \{v_{nm+1}\}$ splits into m components each component isomorphic to P_{3k}. Therefore, the minimum dominating set of G is obtained by taking dominating set from each component together with v_{nm+1}. That is, if $S = \cup_{i=1}^{m} S_i$, where S_i denotes γ-set of i^{th} component, then $S \cup \{v_{nm+1}\}$ will be a minimum dominating set of G. Since any vertex not in $S \cup \{v_{nm+1}\}$ will be adjacent to exactly one vertex in $S \cup \{v_{nm+1}\}$, no proper subset will be dominating set in G. Thus, $\gamma(G) = \frac{m(n-1)}{3} + 1$.

Next, suppose $n \equiv 2(\text{mod } 3)$. Here, we may consider three possible cases. First, assume $m \equiv 0(\text{mod } 3)$. Then $\{v_m, v_{2m}, v_{3m}, \ldots, v_{nm}\}$ will be a dominating set of cardinality $\frac{nm}{3}$. On the other hand, let D be a dominating set in G and assume $v_{nm+1} \in D$. As the vertex v_{nm+1} dominates m vertices, to cover the remaining vertices, at least $m\lceil \frac{n}{3} \rceil$ vertices are necessary. Thus, we must have, $|D| \geqslant m\lceil \frac{n}{3} \rceil + 1$, which is not possible. Hence, $v_{nm+1} \notin D$. This shows that the domination number of G co-incides with that of a cycle. Therefore, $\gamma(G) = \lceil \frac{nm}{3} \rceil$. Next, suppose $m \equiv 1(\text{mod } 3)$. In this case $\{v_1, v_3, v_6, \ldots, v_{nm-1}\}$ will be a dominating set of size $\frac{nm+2}{3}$, i. e., $\lceil \frac{nm}{3} \rceil$. On the other hand, as in the above case, it is easy to observe that $v_{nm+1} \notin D$, for any dominating set D of G. Therefore, $\gamma(G) = \lceil \frac{nm}{3} \rceil$.

Finally, assume $n \equiv 0(\text{mod } 3)$. For any integer $m \geqslant 3$, clearly nm will be a multiple of 3. Further, no dominating set D contains the center vertex v_{nm+1}. Hence, $\gamma(G) = \gamma(C_{nm})$, i. e., $\gamma(G) = \frac{nm}{3}$. $\rhd$

Proposition 1.2. *Suppose $m \geqslant 3$ and $n \equiv 1(mod\ 3)$, then*

$$\gamma_{td}(J_{n,m}) = \begin{cases} \frac{m(n-1)}{3} + 2, & \text{if } m = 3\ ; \\ \frac{m(n-1)}{3} + 1, & \text{otherwise.} \end{cases}$$

$\lhd$ Let $J_{n,m}$ be a Jahangir graph with $m \geqslant 3$ and $n \equiv 1(\text{mod } 3)$. If $m = 3$, then $J_{n,3}$ contains three minimum dominating sets among which two of them having a common vertex. Thus, $\gamma_{td}(J_{n,m}) = \frac{m(n-1)}{3} + 2$. Next, Assume $m \geqslant 4$. Then, $J_{n,m}$ contains three minimum dominating sets. The dominating set $\{3, 7, 11, \ldots, v_{nm-1}, v_{nm+1}\}$ intersects other two sets and hence itself become a transversal dominating set. Therefore, $\gamma_{td}(J_{n,m}) = \frac{m(n-1)}{3} + 1$. $\rhd$

Theorem 1.7. *Suppose $m \geqslant 3$ and $n \equiv 1(mod\ 3)$, then $\gamma(\mathfrak{D}(J_{n,m})) = 2\gamma(J_{n,m})$.*

$\lhd$ Let $J_{n,m}$ be a Jahangir graph with $m \geqslant 3$ and $n \equiv 1(\text{mod } 3)$. Let S be any minimum dominating set of $J_{n,m}$. Then, S dominate the double graph $\mathfrak{D}(J_{n,m})$ except the corresponding vertices of S in the other copy of $J_{n,m}$. Since none of the vertices in S have common neighbor in itself, the minimum dominating set of $\mathfrak{D}(J_{n,m})$ is obtained by adding the corresponding vertices of S. Therefore, $\gamma(\mathfrak{D}(J_{n,m})) = 2\gamma(J_{n,m})$. $\rhd$

Proposition 1.3. *Suppose $m \geqslant 3$ and $n \equiv 1(mod\ 3)$. Then $\gamma_{td}(\mathfrak{D}(J_{n,m})) = \gamma(\mathfrak{D}(J_{n,m}))$.*

$\lhd$ Let $J_{n,m}$ be a Jahangir graph with $m \geqslant 3$ and $n \equiv 1(\text{mod } 3)$. We observe that $J_{n,m}$ contains a unique dominating set, the double graph $\mathfrak{D}(J_{n,m})$ also contains only one dominating set and hence, $\gamma_{td}(\mathfrak{D}(J_{n,m})) = \gamma(\mathfrak{D}(J_{n,m}))$. $\rhd$

Theorem 1.8. *Let $G \cong J_{n,m}$ be a Jahangir graph with $n \equiv 2(mod\ 3)$. Then*

$$\gamma_{td}(G) = \begin{cases} \lceil \frac{mn}{3} \rceil + 2, & \text{if } m \equiv 0\,(mod\ 3); \\ \lceil \frac{mn}{3} \rceil + 1, & \text{if } m \equiv 1\,(mod\ 3); \\ \lceil \frac{mn}{3} \rceil, & \text{if } m \equiv 2\,(mod\ 3). \end{cases}$$

$\lhd$ Let $G \cong J_{n,m}, m \geqslant 3$ be a Jahangir graph such that $n \equiv 2(\text{mod } 3)$. First, we note that dominating set in $J_{n,m}$ arises from dominting set of the cycle C_{nm} and hence any transversal dominating set in $J_{n,m}$ contains at least one vertex from the set $D = \{v_1, v_2, v_3\}$. There are three possible cases here. suppose $m \equiv 0(\text{mod } 3)$, then $nm \equiv 0(\text{mod } 3)$. Thus, $J_{n,m}$ contains exactly three vertex disjoint dominating sets each of cardinality $\frac{nm}{3}$. Therefore $\gamma_{td}(G) \leqslant \frac{nm}{3} + 2$. On the other hand, since any γ-set contains vertex from D, it follows that $\gamma_{td}(G) = 3 + \gamma(H)$, where H is the graph induced by $V(J_{n,m}) - D$. Clearly, $H \cong P_{mn-5}$ and hence, $\gamma_{td}(J_{n,m}) = \lceil \frac{nm}{3} \rceil + 2$. Suppose $m \equiv 1(\text{mod } 3)$, then $J_{n,m}$ contains two vertex disjoint dominating sets. Hence, γ_{td}-set of $J_{n,m}$ is obtained by adding one vertex to the γ-set of $J_{n,m}$. Therefore, $\lceil \frac{mn}{3} \rceil + 1$. Finally, suppose $m \equiv 2(\text{mod } 3)$. As the graph $J_{n,m}$ contains only one dominating set $\{v_1, v_4, v_7, \ldots, v_{mn-4}, v_{mn-1}\}$ and hence itself a transversal dominating set. Therefore, $\gamma_{td}(J_{n,m}) = \gamma(J_{n,m}) = \lceil \frac{mn}{3} \rceil$. $\rhd$

Theorem 1.9. *Let $G \cong J_{n,m}$ be a Jahangir graph with $n \equiv 2(mod\ 3)$. Then $\gamma_{td}(\mathfrak{D}(G)) = 2\lceil \frac{nm}{3} \rceil - \lfloor \frac{m-1}{2} \rfloor + 1$. Further, $\gamma_{td}(\mathfrak{D}(G)) = \gamma(\mathfrak{D}(G))$.*

$\lhd$ Let $G \cong J_{n,m}$ be a Jahangir graph with $n \equiv 2(\text{mod } 3)$. Let S be a minimum dominating set in G. Then, S dominates the double graph $\mathfrak{D}(G)$ except the vertices in the second copy of G corresponding to that of S. Also, the vertex v'_{nm+1} at the center will be adjacent to exactly $\lfloor\frac{m-1}{2}\rfloor$ vertices of S. Hence, the minimum dominating set will be obtained by choosing $|S|-\lfloor\frac{m-1}{2}\rfloor$ vertices from the second copy of G. Therefore, $\gamma_{td}(\mathfrak{D}(G))=2\lceil\frac{nm}{3}\rceil-\lfloor\frac{m-1}{2}\rfloor+1$. Next, since any dominating set in $\mathfrak{D}(G)$ contains the center vertex v_{nm+1}, it follows that any γ-set itself a transversal dominating set in $\mathfrak{D}(G)$. Hence, $\gamma_{td}(\mathfrak{D}(G))=\gamma(\mathfrak{D}(G))$. $\rhd$

Proposition 1.4. *Let $G\cong J_{n,m}$ be a Jahangir graph with $n\equiv 0(mod\ 3)$ and $m\geqslant 3$. Then $\gamma_{td}(G)=\gamma(G)$.*

$\lhd$ Let $G\cong J_{n,m}$ be a Jahangir graph with $n\equiv 0(\mathrm{mod}\ 3)$ and $m\geqslant 3$. For any value of m, we have $nm\equiv 0(\mathrm{mod}\ 3)$ and so G contains unique dominating set $\{v_1,v_4,v_7,\dots,v_{nm-5},v_{nm-2}\}$. Therefore, $\gamma_{td}(G)=\gamma(G)=\lceil\frac{mn}{3}\rceil$. $\rhd$

Theorem 1.10. *Let $G\cong J_{n,m}$ be a Jahangir graph with $m\geqslant 3$. If $n\equiv 0(mod\ 3)$, then $\gamma(\mathfrak{D}(G))=\frac{nm}{3}+1$.*

$\lhd$ Let $G\cong J_{n,m}$ be a Jahangir graph with $m\geqslant 3$ and $n\equiv 0(\mathrm{mod}\ 3)$. Since G contains a unique dominating set $D=\{v_1,v_4,v_7,\dots,v_{nm-5},v_{nm-2}\}$ and the set fails to dominates the corresponding vertices in the second copy of G. Therefore, $\gamma(G)\geqslant\frac{nm}{3}+1$. On other hand, since the center vertex v_{nm+1} is adjacent to every vertex in D, the set $D\cup\{v_{nm+1}\}$ will be a dominating set and hence, $\gamma(\mathfrak{D}(G))=\frac{nm}{3}+1$. $\rhd$

Theorem 1.11. *Let $G\cong J_{n,m}$ be a Jahangir graph with $m\geqslant 3$. If $n\equiv 0(mod\ 3)$, then $\gamma_{td}(\mathfrak{D}(G))=\frac{nm}{3}+2$.*

$\lhd$ Let $G\cong J_{n,m}$ be a Jahangir graph with $m\geqslant 3$ and $n\equiv 0(\mathrm{mod}\ 3)$. From the above theorem, it follows that G conatins exactly two dominating sets having no vertex in common. Thus adding one vertex from a dominating set to other set, the transversal dominating set will be obtained. therefore, $\gamma_{td}(\mathfrak{D}(G))=\frac{nm}{3}+2$. $\rhd$

2. Bounds for $\gamma_{td}(\mathfrak{D}(G))$

Theorem 2.1. *Let G be any connected graph of order n. Then $1\leqslant\gamma(G)\leqslant\gamma(\mathfrak{D}(G))\leqslant\gamma_{td}(\mathfrak{D}(G))\leqslant 2n$. Further, $\gamma(\mathfrak{D}(G))=\gamma_{td}(\mathfrak{D}(G))$ holds if and only if G contains a unique dominating set of size 1.*

$\lhd$ Let G be any connected graph of order n. Since any dominating set of double graph of G dominates G also, it follows that $\gamma(G)\leqslant\gamma(\mathfrak{D}(G))$. Assume $\gamma(\mathfrak{D}(G))=\gamma_{td}(\mathfrak{D}(G))$. On contrary, suppose $\gamma(G)\geqslant 2$ and let S be a γ-set. Then S dominates the double graph $\mathfrak{D}(G)$ except the corresponding vertices of S in other copy of G. Therefore, S cannot be a transversal dominating set in $\mathfrak{D}(G)$, showing that $\gamma(\mathfrak{D}(G))\neq\gamma_{td}(\mathfrak{D}(G))$. Conversly, if G contains a unique dominating set of cardinality one, then $\mathfrak{D}(G)$ contains unique dominating set and so $\gamma(\mathfrak{D}(G))=\gamma_{td}(\mathfrak{D}(G))$. $\rhd$

Corollary 2.1. *Let G be any graph. Then $2\leqslant\gamma(\mathfrak{D}(G))\leqslant\gamma_{td}(\mathfrak{D}(G))$. Further, $\gamma_{td}(\mathfrak{D}(G))=2$ if and only if G is a star.*

$\lhd$ Let G be any graph. First, assume $\gamma_{td}(\mathfrak{D}(G))=2$. From the above theorem it follows that $\gamma(\mathfrak{D}(G))=\gamma_{td}(\mathfrak{D}(G))$ and so G must contain exactly one vertex of degree $n-1$, proving that G is a star. Converse is obvious. $\rhd$

There is no exact relation between $\gamma_{td}(G)$ and $\gamma(\mathfrak{D}(G))$. For example, if G is a star, then $\gamma(\mathfrak{D}(G))=\gamma_{td}(\mathfrak{D}(G))$. Let G be a complete graph of order $n\geqslant 4$, then $\gamma_{td}(G)=n-1>2=\gamma(\mathfrak{D}(G))$. Finally, let G be a path of P_6. Then $\gamma_{td}(G)=5$ but $\gamma(\mathfrak{D}(G))=6$.

Proposition 2.1. *Let G be a connected graph of order $n\geqslant 2$. Then $\gamma_{td}(\mathfrak{D}(G))\leqslant\gamma(\mathfrak{D}(G))+\delta(\mathfrak{D}(G))$.*

◁ Let G be a connected graph of order $n \geqslant 2$. Then, $\delta(G) \geqslant 1$ and let v be a vertex of degree $\delta(G)$. Clearly, any dominating set in $\mathfrak{D}(G)$ must contain either v or a vertex from $N(v)$. Thus, $\gamma_{td}(\mathfrak{D}(G)) \leqslant \gamma(\mathfrak{D}(G)) + |N(v)|$. This proves that, $\gamma_{td}(\mathfrak{D}(G)) \leqslant \gamma(\mathfrak{D}(G)) + \delta(\mathfrak{D}(G))$. ▷

Theorem 2.2. *Let G be any graph. Then $\gamma_{td}(\mathfrak{D}(G)) = 2n - 1$ if and only if G is a complete graph.*

◁ Let G be a connected graph of order n. Assume that $\gamma_{td}(\mathfrak{D}(G)) = 2n - 1$. Then, any subset S' of vertices of order atmost $2n-2$ is not a transversal dominating set in $\mathfrak{D}(G)$. From the minimality of $\gamma_{td}(\mathfrak{D}(G))$, it follows that, $V - S' = \{u, v\}$ is a dominating set in $\mathfrak{D}(G)$. Further, $V - S'$ must contains at least one vertex from each copy of G. Thus, $\gamma(G) = 1$. As the vertices u, v are chosen arbitrarily, each vertex in G must have degree $n - 1$, proving that G is a complete graph. Converse is obvious. ▷

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Someshwarapura R. Nayaka
P.E.S. College of Engineering,
Mandya, Karnataka 571401, India,
Assistant Professor
E-mail: nayaka.abhi11@gmail.com

Puttaswamy
P.E.S. College of Engineering,
Mandya, Karnataka 571401, India,
Professor
E-mail: prof.puttaswamy@gmail.com

K. N. Prakasha
Vidyavardhaka College of Engineering,
P.B. No.206, Gokulam III Stage,
Mysuru 570002, Karnataka, India
Assistant Professor
E-mail: prakashamaths@gmail.com

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ТРАНСВЕРСАЛЬНОЕ ДОМИНИРОВАНИЕ В ДВОЙНЫХ ГРАФАХ

Наяка С. Р.[1], Путтасвами[1], Пракаша К. Н.[2]

[1] Инженерный колледж, Мандья, Карнатака 571401, Индия;
[2] Инженерный колледж Видьявардхаки, Майсур, Карнатака 571401, Индия
E-mail: nayaka.abhi11@gmail.com, prof.puttaswamy@gmail.com,
prakashamaths@gmail.com

Аннотация. Пусть G — произвольный граф. Подмножество S множества всех вершин G называется доминирующим множеством, если каждая вершина, не входящая в S, примыкает, по меньшей мере, к одной из вершин из S. Доминирующее множество S называется трансверсальным доминирующим множеством, если S имеет непустое пересечение с каждым доминирующим множеством минимальной мощности в G. Минимальная мощность трансверсального доминирующего множества называется числом трансверсального доминирования, обозначаемым $\gamma_{td}(G)$. В данной статье рассматриваются специальные типы графов, называемые двойными графами, получаемыми с помощью операций над графами. Мы изучаем новый параметр доминирования для этих графов. Вычисляется точное значение числа доминирования и числа поперечного доминирования в двойных графах некоторого стандартного класса графов. Кроме того, получены некоторые простые оценки для этих параметров в терминах порядка графа.

Ключевые слова: поперечное доминирующее множество, число поперечного доминирования, прямое произведение, двойной граф.

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