# Computer Algebraic Treatment of Complex Word Problems

Heinz Schumann, Weingarten

Planning pays!

**Abstract:** The development of the ability to solve problems is a task of general education schools in the framework of the cultivation of the ability to learn. Part of this ability is systematic planning of solutions of problems. Using the example of complex word problems, it is shown how planning of solutions can be motivated, contextually implemented, and executed by means of computer algebra. Further new possibilities of the treatment of problems result from the modification of once implemented formulations (Ansätze). The work closes with a report on a first practical test during instruction and with aspects of the further exploration of this subject.

**Kurzreferat:** *Behandlung komplexer Textaufgaben mit Hilfe von Computeralgebra.* Die Entwicklung der Problemlösefähigkeit ist eine Aufgabe allgemeinbildender Schulen im Rahmen der Kultivierung der Lernfähigkeit. Ein Teil dieser Fähigkeit ist das systematische Planen von Problemlösungen. Am Beispiel komplexer Textaufgaben wird gezeigt, wie die Planung der Lösung mit Hilfe von Computeralgebrasystemen motiviert, kontextuell implementiert und ausgeführt werden kann. Weitere neue Möglichkeiten der Problembehandlung resultieren aus der Modifizierung bereits implementierter Ansätze. Die Arbeit endet mit einem Bericht über eine erste Erprobung im Unterricht und mit Aspekten zur weiteren Erforschung dieses Gebiets.

**ZDM-Classification:** H30, R20, U70

## 1. Introduction

It is well known that students tend to have little success in working on word problems. The cause is mainly a lack of the capability to comprehend and/or to model the extra-mathematical facts described colloquially in the problem. Additional difficulties are caused by the algorithmic solution of the corresponding mathematical model (cf. e.g. Reusser 1996). The use of suitable computer algebra systems opens up new possibilities for the treatment of word problems in the following manner (cf. Schumann 1995a,b):

- Planning of solutions in context using “word-variables”
- Automatic implementation of plans with individual formulations
- Experimentation with implemented plans of solution (Ansätze):
  Generalisation, specialisation and analogisation of problems via modification of the formulation;
  extensions of the problem by modifications of formulations

*Explanations:* In the framework of the learning of methods (here: of the learning of methods to solve word problems), the planning of solutions is of considerable importance, especially with more complex problems (the reader is reminded of the importance of planning in many areas of vocational and private life; cf. e.g. Reetz et al. 1990). – The quality of the planning is generally decisive for the success of a solution. Although explicit planning of solutions with mathematical problems is emphasised heuristically (cf. Polya 1949; Strehl 1979), it does not take place in the practice of solving (word) problems. In general, students can be observed to follow a run-off tactic (which is not to say that there are no elements of planning present in their minds at all). In this context Kellerer (1935), notes a complete planning of solutions when working with word problems using equations concerning variables.

*Problem:*
*A wine merchant has bought 24 hl of a first, and 35 hl of a second quality of wine for a total price of 9.000 DM. He remembers to have paid 2,50 DM per litre of wine of the first kind. How much does a hectolitre of the second kind cost?*
(with alteration of the numeric values and insignificant textual alterations compared to the original problem)

*Plan of solution:*

$2{,}50\,\mathrm{DM} \cdot 100 = a\,\mathrm{DM}$
$a\,\mathrm{DM} \cdot 24 = b\,\mathrm{DM}$
$9.000\,\mathrm{DM} - b\,\mathrm{DM} = c\,\mathrm{DM}$
$c\,\mathrm{DM}/35 = d\,\mathrm{DM}$

Kellerer writes (translation by the author): “Initially the children are disgusted by the application of this strait-jacket. Some even produce first the minor workings and only construct the path to the solution retrospectively. However, once accustomed to the method, they enjoy it greatly.”

The graphical development and representation of plans of solutions for word problems in the form of *calculation trees* (cf. Meyer et al. 1978) has all but vanished from today’s school books. However, calculation trees prove to be a flexible aid in planning interactive solutions of word problems in tutorial systems (Stüssi 1995). The use of variable names in the form of complete words or phrases in context, still missing in Kellerer’s work, facilitates planning and helps its self-evidence. The precondition for a nomenclature in context and use of “*complete word variables*” is that no manipulations or calculations by hand are needed.

If a plan is available, the use of a suitable auto-solver of a computer algebra system eliminates the customary solution by hand. The advantage is that students are motivated to put effort in the explicit planning of a solution. A simple internal check can be facilitated by interchanging given and wanted quantities; furthermore, all intermediate results can be shown.

There is no need for a renewed implementation of a plan (Ansatz), because it can be duplicated and modified using the options “copy”, “insert” and “overwrite”, which are well known from text editing.

*Generalisation of problems:* By replacing the actual given quantities with suitable names, the students obtain a general formulation. The automatic solution of this formulation implies the solution of the corresponding class of problems. For the student, the terms of the solution clarify the functional dependence of given quantities on the wanted ones.

It is now possible to experiment with a formulation in a multitude of ways. For example, it is possible to use other than the given quantities or to define constraints for some data, according the motto: What happens to ... if ... ? An "operational working" of the problem becomes economical only because calculations by hand are not required anymore. Interchanging given and wanted quantities result in new problems; the original formulation has to be modified correspondingly. In the following the possibilities discussed above are implemented using a realistic word problem.

## 2. An Example ("Fabian problem")

*Problem:*
*Next year, Fabian will need a bicycle for his way to school. He has seen a 21-gear one at the bicycle dealer, which pleases him very much. It costs 495,–DM. In the sporting goods shop he would have to pay 35 DM more for the same bicycle. Fabian's parents will cover a third of the costs. Fabian delivers newspapers and earns 13,50 DM a week. Today, he has gotten the earnings for the past 13 weeks. From that, he has to pay 45,–DM for guitar lessons. He puts aside the rest for the bicycle. How much money does Fabian still have to save until he can buy the cycle at the bicycle dealer?*
(Comment on the formulation of the problem: The problem contains extraneous information. The student is required to recognise it as irrelevant to the solution.) To solve this problem, we follow the instructions for the solution of word problems using an auto-solver and obtain a first formulation, which takes into account one by one the information given in the problem (see Fig. 1).

*A first formulation:*
price = 495 DM,
price/3 = parents,
$13 \cdot 13{,}50\,\text{DM} = \text{earnings}$,
$\text{earnings} - 45\,\text{DM} = \text{rest}$,
$\text{parents} + \text{rest} = \text{credit}$,
$495\,\text{DM} - \text{credit} = \text{savings}$,
Solve for "savings" and the intermediate results:
"parents, earnings, rest, credit".

Using somewhat different words we develop another formulation, a so-called top-down formulation, where we start with the wanted quantities and specify gradually downwards. The advantage of such a formulation is its clear and strict organisation. Therefore we should aim to use this kind of formulation, suitable for documentation, in class. We emphasise that the solver of the problem is completely free in the choice of names for quantities, of the sequence of equations, of the order of sides of equations and of the number of auxiliary variables.

*A second formulation:*
$\text{savings} = 495\,\text{DM} - \text{available}$,
$\text{available} = \text{parental} + \text{own}$
$\text{parental} = 495\,\text{DM}/3$
$\text{own} = \text{earnings} - \text{deductions}$,
$\text{earnings} = 13 \cdot 13{,}50\,\text{DM}$,
$\text{deductions} = 45\,\text{DM}$
Solve for "savings"; auxiliary quantities to be eliminated are:
"available, parental, own, earnings".
(Here it is also possible to solve for all intermediate results and the wanted quantity simultaneously.)

**Instruction for solving word problems à la Polya using an auto-solver**

1. **Comprehend the problem!**
   Do you understand everything in the text?
   If not, take a dictionary or school book and look it up.
   Underline anything of importance in the text. (Use different colours.)
   What is given, what is wanted? Cross out unimportant parts.
   Which conditions are there?
   Do you know a similar problem, which you have worked on already?
2. **Plan the solution! – Find a formulation!**
   What relationships are there between given and wanted quantities?
   Do you know already a suitable plan or are at least parts of it applicable?
   Form all possible equations. Use complete words in order to understand what they designate. Possibly you need to introduce auxiliary variables, for example for intermediate results, to express the given and wanted quantities in terms of equations. Did you take into account all conditions mentioned in the problem?
   Check if your formulation is complete. (Necessarily your formulation must contain as many equations as wanted proper and auxiliary quantities together.)
   Can you find still another formulation for the solution?
3. **Run your plan!**
   Enter your formulation like this: First the set of equations, then the set of wanted quantities and finally the set of auxiliary variables.
   Let the auto-solver solve your formulation.
4. **Check the output for the solution!**
   Is the result meaningful, i.e. does it approximately agree with your expectation from a rough estimate? Otherwise check your formulation. Check your calculation by replacing wanted and given quantities in the formulation and apply the auto-solver to it.
5. **Formulate the result!**
   Write down an answer in the form of a sentence.

Fig. 1

Assuming the mathematical complexity of a word problem is determined by the number of given, wanted and auxiliary quantities and the number and kind of algebraic equations, the Fabian problem in its specific form can be considered of "intermediate" complexity. This problem comprises of only linear equations and only one wanted quantity. However, with five equations concerning five given and four auxiliary quantities, the number of variables is rather large.

We implement both formulations: Input 1 shows our first formulation in a modified syntax of Mathematica 3.0. (In general, the computer algebra of Derive allows a less flexible and more confined implementation of formulations.) As a result we obtain the various amounts in a slightly unfamiliar fashion (output 1). The order of the implementation of the 6 equations is immaterial; there are 6!=720 different possibilities for the input sequence. The so-called special formulations can be autosolved using specific quantities (for example in units of DM, m, kg). (Units are treated as variables during autosolution.) In- and output 2 represents our second formulation (top-down formulation). To perform (internal to the system) a check of the calculation, we replace throughout the formulation 495 DM by "price" and "savings" by 199,50 DM. The quantity to be determined is now the price (input 3). As expected, the result for the price is 495 DM (output 3). If we consider all auxiliary quantities also as wanted quantities, we obtain all possible information (in- and output 4).

```
    {price==495DM,
    price/3==parents,
    13*13.50DM==earnings,
    earnings-45DM==rest,
    parents+rest==credit,
    495DM-credit==savings},
    {savings,parents,rest,earnings,credit}
{savings == 199.5DM, parents == 165.DM,
rest == 130.5DM, earnings == 175.5DM,
credit == 295.5DM}
```

*Input and output 1*

```
    {savings==495DM-available,
    available==parental+own,
    own==495DM/3,
    own==earnings-deductions,
    earnings==13*13.50DM,
    deductions==45DM},
    {savings},
    {available,parental,own,earnings}
savings == 199.5DM
```

*Input and output 2*

```
    {199.50DM==price-available,
    available==parental+own,
    parental==price/3
    own==earnings-deductions,
    earnings==13*13.50DM,
    deductions==45DM},
    {price},
    {available,parental,own,earnings}
price == 495.DM
```

*Input and output 3*

```
    {savings==495DM-available,
    available==parental+own,
    parental==495DM/3,
    own==earnings-deductions,
    earnings==13*13.50DM,
    deductions==45DM},
    {savings,available,parental,own,earnings}
{savings == 199.5DM, available == 295.5DM,
parental == 165.DM, own == 130.5DM,
earnings == 175.5DM}
```

*Input and output 4*

*Generalisation of the problem:*

Now we can continue, for example using the second formulation (cf. input 2), to develop a formulation for the general problem represented by the example problem at hand. To do this, we replace the concrete quantities by variable names (input 5). The solution (output 5) shows the functional dependence of the wanted quantity (savings) on the five given quantities (price, part, deductions, wage, weeks). Generalising the first formulation leads to a corresponding result. At a glimpse we recognise how the amount to be saved is made up from the various given quantities and basic operations. We can improve the structure of the result by hand:

$$\text{savings} = \text{price} + \text{deductions} - \left(\frac{\text{price}}{\text{part}} + \text{wage} \cdot \text{weeks}\right).$$

The terms of the solution for intermediate calculations are shown in output 6. Any problem of this type can now be solved by specification of the general formulation. All that is needed is to insert the assignments for the corresponding values (input 7). Likewise, the assignments can be included anywhere in the general formulation.

```
    {savings==price-available,
    available==parental+own,
    parental==price/part,
    own==earnings-deductions,
    earnings==weeks*wage},
    {savings},
    {available,parental,own,earnings}
```

$$\texttt{savings} == \texttt{price} - \frac{\texttt{price}}{\texttt{part}} + \texttt{deductions} - \texttt{wage weeks},$$

*Input and output 5*

```
    {savings==price-available,
    available==parental+own,
    parental==price/part,
    own==earnings-deductions,
    earnings==weeks*wage},
    {savings,available,parental,own,earnings}
```

$$\{\texttt{savings} == \texttt{price} - \frac{\texttt{price}}{\texttt{part}} + \texttt{deductions} - \texttt{wage weeks},$$

$$\texttt{available} == \frac{\texttt{price}}{\texttt{part}} - \texttt{deductions} + \texttt{wage weeks},$$

$$\texttt{parental} == \frac{\texttt{price}}{\texttt{part}},$$

$$\texttt{own} == -\texttt{deductions} + \texttt{wage weeks},$$

```
earnings == wage weeks}
```
*Input and output 6*

```
    {price==495,part==3,weeks==13,
    wage==13.50,
    deductions==45,
    savings==price-available,
    available==parental+own,
    parental==price/part.,
    own==earnings-deductions,
    earnings==weeks*wage},
    {savings},
    {available,parental,own,earnings}
savings == 199.5
```
*Input and output 7*

We will now consider questions which may result from the original problem and lead to new problems. The new problem is solved by modifying the formulation of the original problem.

A first *problem extension*:
*How many weeks would Fabian still have to work to be able to pay the full price at once, taking into account the part paid by his parents?*

First we solve this problem experimentally by overwriting and increasing the number of weeks (input and output 8: for 27 weeks 10,50 DM remain; input and output 10: 28 weeks earn already 3 DM over).

We want to know the exact answer; to that we add the condition savings = 0 to the formulation (input 8 and 9, respectively), omit weeks = ... and solve for weeks (input 10). The rounded result reads: 27,8 weeks (output 10), that is 27 weeks and 4 days, assuming 5 working days per week.

```
    {price==495,part==3,
    weeks==27,wage==13.50,deductions==45,
    savings==price−available,
    available==parental+own,
    parental==price/part,
    own==earnings−deductions,
    earnings==weeks*wage},
    {savings},
    {available,parental,own,earnings}
savings == 10.5
```
*Input and output 8*

```
    {price==495,part==3,weeks==28
    wage==13.50,deductions==4,
    savings==price−available,
    available==parental+own,
    parental==price/part,
    own==earnings−deductions,
    earnings==weeks*wage},
    {savings},
    {available,parental,own,earnings}
savings == −3.
```
*Input and output 9*

```
    {savings==0,
    price==495,part==3,deductions==45,
    wage==13.50,
    savings==price−available,
    available==parental+own,
    parental==price/part,
    own==earnings−deductions,
    earnings==weeks*wage},
    {weeks},
    {available,parental,own,earnings}
weeks == 27.7778
```
*Input and output 10*

A *problem analogisation:*
*Fabian would like to buy a video camera; he notices a sale offer of 1.999,– DM. Again the parents want to contribute a third of the price. On the side he earns 20,– DM a week and he has worked already for 36,5 weeks. From these earnings he has to repay 125,– DM to his sister.*

Specialising the general formulation in input 5 or overwriting input 8 we obtain an amount of 727,67 DM, which Fabian would yet have to save (input and output 11).

```
    {price==1999,part==3,
    weeks==36.5,wage==20,deductions==125,
    savings==price−available,
    available==parental+own,
    parental==price/part,
    own==earnings−deductions,
    earnings==weeks*wage},
    {savings},
    {available,parental,own,earnings}
savings == 727.667
```
*Input and output 11*

A *problem* essentially *analogous* and in a *very different* context reads as follows:
*A metal processing company in the new states of Germany needs to invest 495000 ECU for a machine tool to stay competitive. The EC covers a third. For the purchase of the machine, the company was able to build reserves of 13500 ECU for each of the past 13 months. However, for structural modifications required to install the machine 45000 ECU have already been deducted. How much money does the company need to raise with their bank to be able to finance the purchase?*

Other than names, the formulations for this problem are the same as those of the "Fabian problem".

Further *problem extensions:*
*Which video camera could Fabian afford, if he could count on the same available means and his grandmother promises an additional 100,– DM on occasion of his birthday?*
*Thus grandmother saves Fabian 100,-DM.*

The solution is given in input and output 12.

```
    {savings==100,part==3,weeks==36.5,
    wage==20,deductions==125,
    savings==price-available,
```

```
available==parental+own,
parental==price/part,
own==earnings-deductions,
earnings==weeks*wage},
{price},
{available,parental,own,earnings}
price == 1057.5
```

*Input and output 12*

*Fabian's uncle Arthur, a video fan, also wants to contribute to the purchase of the camera, however, for the time being he leaves the amount of his contribution undetermined.*

To solve this problem, we have to treat "savings" as a variable and output the price as a function of savings (input and output 13; given the conditions, the price is a linear function of the savings).

```
{part==3,weeks==36.5,
wage==20,deductions==125,
savings==price−available,
available==parental+own,
parental==price/part,
own==earnings−deductions,
earnings==weeks*wage},
{price},
{available,parental,own,earnings}
price == 907.5 + 1.5 savings
```

*Input and output 13*

*Fabian needs to reckon with deductions of an amount yet unspecified.*

Solution in input and output 14.

```
{part==3,weeks==36.5,wage==20,
savings==price−available,
available==parental+own,
parental==price/part,
own==earnings−deductions,
earnings==weeks*wage},
{price},
{available,parental,own,earnings}
price == 1095. + 1.5 savings − 1.5 deductions
```

*Input and output 14*

If *Fabian is completely in the dark about concrete values* for the conditions of a purchase, output 15 shows him how the price depends on the corresponding variables.

```
{savings==price−available,
available==parental+own,
parental==price/part,
own==earnings−deductions,
earnings==weeks*wage},
{price},
{available,parental,own,earnings}
```

$$\text{price} == \frac{\text{part (savings} - \text{deductions} + \text{wage weeks)}}{1 + \text{part}}$$

*Input and output 15*

As with the price, the general formulation (cf. input 5) can be solved for other given variables. We obtain:

$$\text{part} = \frac{\text{price}}{\text{price} + \text{deductions} - (\text{savings} + \text{wage} \cdot \text{weeks})}$$

$$\text{weeks} = \frac{\text{part}\,(\text{price} + \text{deductions} - \text{savings}) - \text{price}}{\text{part} \cdot \text{wage}}$$

$$\text{wage} = \frac{\text{part}\,(\text{price} + \text{deductions} - \text{savings}) - \text{price}}{\text{part} \cdot \text{weeks}}$$

$$\text{deductions} = \text{savings} + \frac{\text{price}}{\text{part}} + \text{wage weeks} - \text{price}$$

These results of modifications of a formulation give rise to corresponding outfits of "Fabian problems".

*Remark:* There are $\binom{6}{5} = 6$ possible formulations of the problem, where each allows to specify 5 of the 6 variables (price, part, weeks, wage, deductions, savings) to determine the value of the 6th variable. Furthermore, the uncertainty of zero, one, two, ..., five of the remaining variables results in $\binom{5}{0}+\binom{5}{1}+\binom{5}{2}+\binom{5}{3}+\binom{5}{4}+\binom{5}{5} = 2^5 = 32$ additional formulations of the problem containing zero, one, two,..., five variables in the terms of the solution, so that we are left with a total of 192 formulations of the problem.

## 3. A first practical test of the concept in class

In grades 8 and 9 of the *German Middle School* (a total of 60 students) formulation oriented solution of word problems was tested in the framework of a class experiment lasting only two hours. A computer (including data display) was used only as a means of demonstration. Teamwork was included in the experiment to improve social skills.

*Plan of progression:*

- During a teacher-student conversation the formulation of a word problem is elaborated without calculations or transformations of terms.
- The teacher shows an implemented formulation and runs it.
- The solution is discussed in a teacher-student conversation.
- The teacher stimulates proposals for modifications of the formulation and implements such proposals.
- In teamwork students themselves develop formulations for a further given problem. Representatives of the teams explain results (on transparency) to the class. Teacher and students implement formulations judged correct. The solution is discussed within the class; generalisation and modification of the formulation etc. follow suit.

Using a questionnaire consisting of ten intuitive questions, students were asked to express their opinion about this kind of instruction. In the following we report on the result of this line of questioning, dependent on the variable "Is mathematics fun?" (compare the citation of Kellerer in the introduction of this work). We evaluate and interpret the corresponding statistical table (Table 1).

| Answer to question: | Is mathematics fun? Yes: Yes | Yes: No | No: Yes | No: No |
|---|---|---|---|---|
| 1) Do you like to work with computers? | 63% (97%) | 2 % (3%) | 32% (90%) | 3% (10%) |
| 2) Do you think today's mathematics lesson was interesting? | 50% (77%) | 15% (23%) | 17% (48%) | 18% (52%) |
| 3) As a teacher, would you use this form of instruction too? | 45% (69%) | 20% (31%) | 20% (57%) | 15% (43%) |
| 4) Did you have some success of learning during this double lesson? | 45% (69%) | 20% (31%) | 13% (38%) | 22% (62%) |
| 5) Do you have difficulties solving word problems? | 57% (87%) | 8% (13%) | 33% (95%) | 2% (5%) |
| 6) Do you think it is reasonable to let the computer do the calculations in fact problems? | 40% (62%) | 25% (38%) | 28% (81%) | 7% (19%) |
| 7) Would you like to enter the problems into the computer yourself? | 45% (69%) | 20% (31%) | 28% (81%) | 7% (19%) |
| 8) Would you like to use the computer to solve problems during a class test? | 45% (69%) | 20% (31%) | 32% (90%) | 3% (10%) |
| 9) Do you believe that the computer eases the solution of word problems? | 32% (49%) | 33% (51%) | 23% (67%) | 12% (33%) |
| 10) Is it difficult for you to develop a general formulation (only containing variables) from a special formulation (containing given values)? | 23% (36%) | 42% (64%) | 20% (57%) | 15% (43%) |

Given in brackets are relative percentages

Table 1

Students having and not having fun with mathematics equally enjoy working with the computer (question 1). At any rate about half of the students not having fun find this kind of instruction interesting (question 2) and would use it themselves as instructors (question 3). Students having fun with mathematics answer significantly affirmative to both questions. The self-estimated success of learning is significantly different in both groups and of opposite trend (question 4). Both groups of students encounter equal difficulties with word problems (question 5). In question 9 the majority of the group of students not liking mathematics hopes that the computer will help with the solution of word problems, whereas the other group shows an indifferent opinion. The answers to questions 6 and 8 can be interpreted as follows: Students not having fun with mathematics show a significant tendency to favour the use of computers for the solution of formulations and for the solution of problems in class tests, presumably in the hope of reducing their past failures. Answers to question 7 show the same trend, whereas in connection with question 1 one would expect the answers to be more balanced between the groups. Answers to question 10 show that about two thirds of the students having fun with mathematics do not consider it difficult to generalise a formulation. However, the other group shows a rather indifferent assessment.

Further investigations are needed to clarify whether the students' expectations will be fulfilled.

Some more subjects for qualitative and quantitative research:

- Interactive use of computer algebra during implementation and modification of formulations.
- Comparison of different plan-oriented methods to solve word problems (e.g. the method of a solution tree versus the method of equations)
- Comparison of tutorial versus tool supported solution of word problems etc.

We could also investigate which general and special feedback effects arise due to a computer algebraic treatment of word problems, e.g. during the formulation oriented modelling process. We could assume the following procedure:

***Modelling process during the solution of (mathematical) word problems***

(1) *Comprehension of the text*, i.e.:
- Understanding in context
- Understanding actions
- Action – Reaction – Understanding
- Logical comprehension
- Quantitative comprehension

*Gain of knowledge:*
Registration and comprehension of all information given in and with the text,
Classification of the subject area related to the problem

(2) *Organisation of the text,* i.e.:
- Sort information systematically and clearly
- Check information for completeness
- Exclude irrelevant information
- Logical, spatial and temporal organisation of relevant facts

*Gain of knowledge:*
Given and wanted quantities,
Conditions for given and wanted quantities

(3) *Development of the formulation,* i.e.:
- Use heuristic strategies (e.g., work forwards and backwards within the problem related subject area)
- Introduce auxiliary quantities
- Form terms including signs for operators
- Use formulas and rules of the subject area

*Gain of knowledge:*
- Equation, inequality (to solve for ... ),
- Systems of equations, inequalities (to solve for ... ),
- Functional equations (to investigate for ... )

- Computer algebraic treatment of word problems opens up a new area of research within the field of didactics of computer supported instruction of mathematics.

**4. References**

Kellerer, H. (1935): Bemerkung zur Lösung von Textaufgaben (Remarks on the solution of word problems). – In: Zeitschrift für mathematisch und naturwissenschaftlichen Unterricht aller Schularten, pp. 342/343

Meyer, H.-G. et al. (1978): Neues Mathematisches Arbeitsbuch, Band 1. – Frankfurt/M.: Diesterweg

Polya, G. (1949): Schule des Denkens – Vom Lösen mathematischer Probleme (How to solve it?). Bern: A. Francke

Reetz, L.; Reitmann, T. (Ed.) (1990): Schlüsselqualifikationen: Fachwissen in der Krise? (Key qualifications: special knowledge in crisis) – Hamburg: Feldhaus

Reusser, K. (1996): Textverständnis, Situationsverständnis und Mathematisierung beim Lösen von Textaufgaben (Text comprehension, fact comprehension and mathematical modelling while solving word problems). – In: Weinert, F. E. & Helmke, A., Entwicklung im Grundschulalter (Development in primary level). Weinheim: PVU

Schumann, H. (1995a): Ansatzorientiertes Lösen komplexer Algebra-Aufgaben mit Computeralgebra (Formulaic solution of complex problems using computer algebra). – In: Beiträge

zum Mathematikunterricht 1995. Hildesheim: Franzbecker, pp. 420–423
Schumann, H. (1995b): Ansatzorientiertes Lösen komplexer Algebra-Aufgaben mit Computeralgebra (Formulation oriented solution of complex algebra problems using computer algebra). – In: Mathematik in der Schule Vol. 33 (No. 6), pp. 372–382
Schumann, H. (1997): The use of computer algebra in the formulaic solving of complex algebraic problems. – In: International Journal of Mathematical Education in Science and Technology Vol. 28(No.2), pp. 269–287
Strehl, R. (1979): Grundprobleme des Sachrechnens (Fundamental problems factual calculation). Freiburg i. Br.: Herder
Stüssi, R. (1995): WATGRAF für Windows Version 1.0. – Büren a.A
Wolfram, S. (1991): Mathematica – A System for Doing Mathematics by Computer. – Redwood City/CA: Addison-Wesley

This work is part of a communication given by the author to the Pedagogical Institute of the University of Zürich on 30.10.1996

**Author**
Schumann, Heinz, Prof. Dr., Pädagogische Hochschule Weingarten, Fak. III Mathematik/Informatik, D-88250 Weingarten. schumann@ph-weingarten.de

## Vorschau auf Analysethemen der nächsten Hefte

Für die Analysen der Jahrgänge 29 (1997) und 30 (1998) sind folgende Themen geplant:
– Analysis an Hochschulen
– Mathematik in der Ingenieurausbildung
– Fächerübergreifender Unterricht
– Mathematik und Friedenserziehung.

Vorschläge für Beiträge zu o.g. Themen erbitten wir an die Schriftleitung.

## Outlook on Future Topics

The following subjects are intended for the analysis sections of Vol. 29 (1997) and Vol. 30 (1998):
– Calculus at universities
– Mathematics and engineering education
– Cross curricular activities
– Mathematics and peace education.

Suggestions for contributions to these subjects should be addressed to the editor.