Can Mathematics Educate for Peace?

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Abstract: In this paper we simulate a discussion with our students, in which we alternate specific mathematical issues (as the PERT (Program Evaluation and Review Technique) in 1.2, a discrete dynamical system in 2.1 and fractals in 2.2) that can be applied in situations such as the evolution of populations (2.1) or other well-known contexts such as the economic equilibrium theory or the ecological systems. The students are given occasions to deepen both the meaning and use of the studied model.

We emphasize those cases where the model leads to analyses of behaviour depending on parameters whose values highlight stable states of dynamic systems (regions of peace) and regions where control over the system evolution is lost, and where the system passes from bifurcations to chaos (regions of war).

Kurzreferat: *Kann Mathematik zum Frieden erziehen?* In diesem Beitrag simulieren wir eine Reflexion mit unseren Studenten, in der wir spezielle mathematische Themen (wie PERT (Program Evaluation and Review Technique) in 1.2, ein diskretes dynamisches System in 2.1 sowie Fraktale in 2.2) ansprechen, die in der Populationsdynamik (2.1) oder anderen bekannten Zusammenhängen wie ökonomische Gleichgewichtstheorie oder Ökosysteme angewandt werden können. Dabei wird den Studenten Gelegenheit gegeben, über Bedeutung und Anwendung der untersuchten Modelle zu reflektieren.

Insbesondere werden solche Fälle betrachtet, bei denen das Modellverhalten in Abhängigkeit bestimmter Parameter zu analysieren ist. Die Werte dieser Parameter bestimmen stabile Zustände dynamischer Systeme (Friedensregionen) und Regionen, in denen die Kontrolle über die Evolution des Systems verloren geht und wo das System von Bifurkationen ins Chaos übergeht (Kriegsregionen).

ZDM-Classification: A40, I90, M40

1. Reflecting with our students

Our contribution is actually a consideration we would like to convey to our students, both the ones we had, and those who will listen to us. Open letter or hypothetical seminar, the youth are the ideal addressees of our ideas and hopes (Giannetti/Scarafiotti, 1995).

We can look at the history of mathematics both from an internal and an external point of view. We will thus become aware of the internal dialectic of a self-generated development, a history of ideas implying that "even in times of great political and social perturbations, it is the spiritual things – in the French meaning of the expression – that count the most ..." (Boyer, 1968). But, at the same time, we will also become aware of an unceasing dialectical relationship between science and society, where the connection between scientific theory and practical applications, economic production and ideology, philosophy and social arrangement shows itself to be complex and articulate. We wish to dwell, together with our students, on some of the knots in this plot: they are highlighted by the occasions in which the history of mathematics linked to mankind's history, alternating between war actions and peace situations. Robert Oppenheimer, in a conference in 1953, said that every great discovery partakes in the world of beauty, and we trust knowledge to be good in itself. But it is also an instrument, an instrument for those who will come later ...; it's an instrument for technology, for practical activities, and for mankind's fate. So it is for us as scientists, so it is for us as men.

1.1 A glance at the history of mathematics

From Archimedes' lever to Heron's simple machines theory, such practical need, put forward by the society, is already present in ancient mathematics, though it is much less felt than in later times, when natural science becomes a main developing factor for technique. Winch, screw, siphon, and other instruments constructed starting from these ones, were known and used, but both Archimedes and Heron, as well as other ancient mathematicians, underestimated these "useful" realizations, according nobility and beauty only to the "artes liberales" (Holton, 1978).

Ever since then, there has been a single field in which the available technology was applied, i.e. warfare. Powerful ballistic machines were conceived for siege and defence strategy, though peaceful applications never seem to have had any advantage from them.

The fifteenth and sixteenth centuries saw the training, in various subjects, of a class of technicians, who did not always devote themselves personally to specifically scientifical issues, but who induced other people to do so. These were the great Renaissance artists – architects, sculptors and painters. Canals, dams and earthworks were built together with imposing cathedrals and palaces: all this required new instruments and devices. Mathematics was inevitably recognized as indispensible, thanks not only to mechanics, but also to civil and military architecture, based on rigorous techniques (Dijksterhuis, 1961).

The French Revolution marks the beginning of a different presence of mathematicians in society, and a momentous change in the social role of mathematics.

Mathematicians took an active part in revolutionary events, by offering their skills to the political power. Their knowledge was resorted to in times of danger for the new institution, to build defense works and study new war instruments. The greatest French mathematicians of the time were appointed to teach in the most important, newly established military academies, and some of them even took active part in the revolutionary, and then in the "restored", government (Bottazzini, 1980).

Our century witnesses a great number of mathematical applications to peaceful purposes, as well as warlike ones – unfortunately, as we all know.

Before going into a more detailed analysis of some examples, it is worth giving a short account of two of the most important knots in the above mentioned plot of ideas.

The strategy game theory is the basis for modern mathematical economy, and it has also influenced decision theory, though to a limited extent.

All this research took place in the United States during the World War II, clearly pursuing warlike aims; but, as Conolly said "if one substitutes for the word 'submarines' in the phrase 'search for submarines' alternativo words such as 'minerals', 'lost aircraft', 'new markets for an industrial product', 'victims of kidnapping', or 'a lost key', it is easy to perceive that the scope of the theory is not confined to a purely military context".

Number theory, in all appearances completely useless, has become, on the contrary, the basis for modern security systems. It certainly provides the instruments which are used to control the hundreds of nuclear missiles which have proliferated in this second afterwar. But it is not only the military who require their communications to be made secure by encryption techniques. There are also commercial and political reasons for ensuring that information is secret and secure. In short, the realization of a cipher system (Devlin, 1988) is based on the "near-impossibility" of factoring into its primes a very large number (say of the order of 100 digits). In fact, message encryption corresponds to multiplication of two large primes, decryption to the opposite process of factoring. As there is no quick method of factoring large numbers, it is practically "impossible" to recover the deciphering key.

1.2 Operations research (O.R.) – the PERT

Studies in O.R. began with World War II. The British military command, followed by the American one, summoned in a great number of scientists to work out a proper method to deal with strategy and tactics approaches. Researches on "military operations" were carried out, whose positive outcome should be arrived at with a minimum amount of expenses for equipment and "men".

These were the first researchers in O.R., who contributed to advise the U.S. Navy on the conduct of antisubmarine patrols.

After the success in the military field, O.R. became of interest for "peaceful" industrial applications. A specific example is the PERT (Program Evaluation and Review Technique).

It was developed (1958–59) in order to measure and check the progress stages of the project for the realization of the Polaris rocket. The application of the PERT technique allowed an effective coordination for thousands of contractors and industries partaking in the Polaris programme, bringing the conclusion of the project two years forward, compared with times scheduled at its start.

Private American industry, and world industry soon afterwards, has then introduced the PERT as a guide in projects developing thanks to an optimal coordination of various activities. Examples are: building programmes, computer routines, complex devices maintenance planning.

One of the first purposes of PERT is to determine the probability of realizing a project in a given time; the PERT allows the identification of those activities on which the "greatest effort" must be made, in order to keep the progress within the scheduled times, on pain of compromising the whole project.

But what is PERT? It is a net-like technique: the issue is shown on a directed network, drawing up the analysis of the project devoloping times.

Let us see a simple example: suppose we have to carry out a job consisting of three elementary operations a, b, c; we know that operation b must follow a, while operation c is independent of the other two. We know the realization times for each activity: a, $t = 5$ (weeks); b, $t = 6$ (weeks); c, $t = 10$ (weeks). What is at issue is the "least" possible duration of the whole job.

A PERT network is realized: the nodes are the events, the branches are the activities linking an event to another one, following in time; event ① is the work start, event ③ its conclusion; the network takes into account the priority among the activities, and every branch is labeled with a number showing its duration (see Fig. 1).

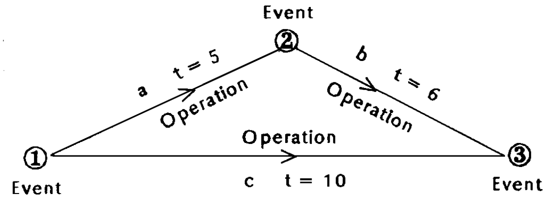

Fig. 1

Therefore (5+6) weeks are necessary for the whole work; if useful, operation c can be delayed one week, without prolonging the total realization time. Vice versa, the activities a and b are called "critical", because a delay in them prolongs the project realization time.

In fact, one of the points at issue in a PERT network is the identification of those critical activities requiring particular resources, not to miss the "appointments" of the project conclusion.

This is, in our opinion, a cue toward the realization of projects involving "high diplomacy for peace", in search of the optimum for men, within technically controlled developing times. We have a good example of this at "Politecnico di Torino": the ISF (ingegneri senza frontiere – engineers without frontiers) association.

2. Can mathematics educate for peace?

To answer this question we have to recall different definitions of peace, leading also to different ways to face the peace education problem in school.

For example, Johan Galtung, one of the most important exponents of the so-called Peace Research, thinks that the crux of the matter is the distinction between "negative peace" and "positive peace". The first one means simply the lack of war or personal violence. On the contrary, positive peace concerns the lack of structural violence that is expressed in social injustice, exploitation and denial of rights.

This way, peace means "full realization of any human right". Now we can easily relate the "positive" definition of peace with the mathematical concept of balance.

If peace is balance and stability in the countries of the world, if war stems from upsetting unsettled balances, then we can identify and examine with our students situations to be interpreted as models for the stability or instability of the system "universe". In the examples we are going to give, what must be "read" is the researching care on the breaking points, since the purpose is that of finding the parameters governing the state of the system, the determination of their values, up to the recognition of the critical cases.

In other words, the teaching context implies the following procedure: a modeled situation is interpreted as a real phenomenon and then the students are stimulated to construct examples for peace situations and war situations.

2.1 Study of a discrete dynamical system on a range interval I

Given I, closed interval of $\mathbb{R}$, let f be an iterative application from I into I, so that $f(I)$ is strictly included in I; the pair $\{I, f\}$ is termed a dynamical system on I. Given $x_0 \in I$, the set

$$\{x_0, f(x_0), f^2(x_0), ...f^n(x_0)\}$$

is called the orbit of x_0 relative to f. Studying a dynamical system, consists, above all, in dealing with two main questions:

– defining the minimal invariant sets;
– studying their attractivity and stability.

If we analyze a dynamical system depending on a control parameter r, we can learn to discuss the above mentioned problems with reference to r. If we consider the dynamical system on $I = [0, 1]$ defined by $f_r(x) = r \times (1 - x)$, we find that the acceptable parameters for the dynamic system fall within the range [0,4], as the maximum of $f_r(x)$ is $P(1/2, r/4)$.

Still, upon studying this system, the existence of a critical value of r, $r = 1 + \sqrt{6}$, is discovered, for which the path branches off; then, with $r = 3.57$, further branching takes place, until, with $r > 3.8284$, the "chaotic" region is reached, owing its name to the swift diversification of the orbit, in connection with x_0 showing just a small difference between each other.

Through a strong simplification, we can conclude that the given example highlights how the basic notion – parameter r falling within a well-defined range – is insufficient to assure a stable state for the phenomenon.

The function $f_r(x)$ is known as "logistic iterator" in mathematical models for the increase of rate r in a population. More properly, the mathematical model for a logistic iterator is

$$g_r(x) = r \times (1 - x/k)$$

where k is the carrying capacity parameter. (See also, Stein, Logistic growth as a problematic example of mathematical model building, in ZDM Vol. 27(Feb. 1995), p. 1–5). Moreover, by studying $f_r(x)$ it is possible to discover that there is a well-defined path which leads from one state into the other state with parameter $r > 3$. The bifurcation phenomena can have very important implications for management of the natural population, be it pasture, fish, whales, insect pests, or human parasitic disease. We take this mathematical model as a metaphor: it is only a question of considering the peaceful living together of nations as a system whose balance depends on a large number of factors, and whose "controllability" we can think to study. On this subject G. Coyle's (1981) essay "A model of the dynamics of the third world war" is very curious: it presents "hypothetical land conflict in Europe, analysed with mathematical methods". The purpose is incorporating the factor of control in the scenario of war, and then the author examines the prospects for the transfer of system dynamics "technology" to defense analysis.

2.2 Fractals

Mandelbrot (quoted in Devlin, 1988) studied the dynamical system

$$x_0 \in \mathbb{C} \qquad x_{n+1} = {x_n}^2 + c \qquad \text{with} \quad c \in \mathbb{C}$$

in the set of complex numbers; already in the case $c = 0$ it is found that x_0 "controls the system", meaning that if $|x_0| < 1$ then zero is attractor for the paths, while if $|x_0| > 1$ then zero is attractor to infinity; lastly, if $|x_0| = 1$ then the path belongs to the unit circle centered at zero.

The dynamical systems in the scope of complex numbers, as well as the dynamical systems on real ranges, are studied with reference to the value of the involved parameters. For example, for $x_{n+1} = {x_n}^2 + c$, $c \in \mathbb{C}$, if $c = 0.31 + 0.04i$ (i.e. it is "just beyond" the real axis by 4.10^{-2}) (Devlin, 1988), the boundary separating the region governed by the attractor and the one governed by infinity is fractally deformed.

The fractal deformation of the boundary gives rise to difficult identifications of the features of the set whose boundary we consider.

The French mathematician Gaston Julia developed much of the theory of the so called Julia sets – as the boundary of a set of points in $\mathbb{C}$ whose orbits do not converge to infinity – in 1915, while he was an in-patient in an army hospital, recovering from the wounds he had received in war. It was not until more than 60 years later that Adrien Douady and John H. Hubbard developed new methods to deepen the implications of Julia sets, using an analogy from electrostatics. The beauty of Douady and Hubbard's work lies in the fact that the potential of any connected set, like the unit circle centered at zero, can be interpreted as a particular polar coordinate system for the escape set, i.e. the set of points for which the iteration escapes (Peitgen et alii, 1992).

In these models, the boundary can be interpreted as the borderline between two conditions. In order to get over "conflictual" situations, a map of the *situations* is required, which main elements are their *boundaries,* the word having, clearly, not only a geographical meaning!

On this subject it's interesting to note that the title of a book dedicated to computer modeling of chaos fractals is just "Exploring the Geometry of Nature" (Rietman, 1989).

It would be interesting for the students to consider the various meanings of the word "boundary" (Benveniste, 1969). Among these, a boundary is a place of meeting, crossing and exchanging, the borderline between different areas of influence. It can be defined as *differentiation,* bringing order in a situation where a lack of differentiation amounts to chaos. But it is also a form of *relation;* as such, it allows to face and discover each other, to cooperate or be opposed, in a never-closing interface.

A very nice little example can be drawn from the last short story in the "Cosmicomiche" by Italo Calvino: "The Count of Montecristo". An invisible line binds and divides Dantès and the Abbot Faria, who share a wish to escape. But they are moved by opposite purposes that involve them in a perpetual closing up and moving away from each other and towards the way of escape, being separated from the external world by a barrier that seems to "grow around them, and the longer they stay enclosed, the farther they move apart".

Such an example might show the students the need for a punctilious attention to maintain a balance situation, as well as a careful analysis and the constant check of the parameters keeping a system in a condition of "peace".

3. Conclusions

Let us go back to the starting question: can mathematics educate for peace? It is not easy to give an answer, the possible replies being so many. The question appears to be even more acute in our days, in consideration of the ethical problems raised by scientific progress, the social responsibility of men of learning, the need to cut out the distance between scientists and citizens, the necessity to make the communication among the various social subjects more transparent. G. H. Hardy, in his famous "A Mathematician's Apology" (1940) tries to solve the problem by "splitting" mathematics into *real* mathematics, the one studied by *real* mathematicians, and the mathematics that he calls *superficial.* This latter is *useful* and *beneficial,* while the former is beautiful but *useless, unharmful* and *innocent;* but, above all, he claims, the real mathematics has no effects on war, whereas the superficial mathematics can be widely employed in war, it can even foster it, by making the war *modern, scientific,* total. Hardy also adds that, when the world goes mad, a mathematician can find in mathematics a matchless narcotic.

We have no certain answers, but it is possible to think of peace as a balance between economic systems, culture development, integration between ethnic groups, and this requires man's faith in its existence.

Mathematics can produce a culture of peace, opening before us scenarios of controlled, or at least controllable, casuality, and mathematics teaching, though unable to give us the certainty of peace, can however concede us the hope of being peace operators, as well as the ability to work for its protection. Once again, it is the mathematician who takes the floor; he, like everyone else, according to what Jonas (1979) tells us, has the *duty toward the future:* the *responsibility* in defending what has "ever and forever" been irreplaceable in the history of mankind and in suggesting a hope for human beings.

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