Peirce and worse

Ormell, Christopher:

The Peircean applicability of mathematics

Mathematics used to tease-out the synoptic, predictable implications of practical and theoretical hypotheses

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The argument of this book is developed around these four interlocking theses (pp. 1–2):

1) Mathematics is a science of "possibilities" drawing unavoidably upon hypotheses.
2) The American philosopher C. S. Peirce (1839–1914) had been the first to realize this.
3) The applications of mathematics to the physical world (hereafter, "applications"), inspired by the previous thesis, should lie at the core of its understanding and teaching.
4) This approach vindicates the view of Ludwig Wittgenstein (1889-1951) that the use of language is necessary to grasp its meaning.

The text "represents a predominantly *philosophical* exercise" (p. 2), intermixed at times with some modern examples of physical situations amenable to mathematical treatment. It also relies upon many assumptions about the historical development of mathematics. This review will ruminate over the same ground, and consider its educational utility.

I

The author sees his third thesis as revolutionary, overthrowing the long-standing prejudice for pure mathematics, some of whose "most eminent" practitioners acted "like unbelievably rich millionaires, who had no need to 'prove' anything about their applicative wealth" (p. 3). This snobbery has lasted "for about twenty-five centuries" (p. 3; compare p. 25), and constitutes the "habitual view of mathematics" (pp. 29, 3).

Even those with only a smattering of the history of mathematics will sense a gross over-simplification here; and it pervades the book throughout. As the author himself notes on p. 52, from and including antiquity applications have motivated most mathematics, in all its various and developing branches; geometry and mechanics have been especially prominent. Have mathematicians put forward a double face for all these centuries, hiding the mass of their work behind pure fig leaves? Of course not. The rise of pure mathematics became prominent only in the 19th century; the author suggests "after 1830" (p. 140), but I find the acceleration to have occurred from the 1860s onwards, correlating positively both with the rapid increase of professionalisation of mathematics at university level, and also with the great rise of Germany (Grattan-Guinness 1997a, esp. Chapters 11–13). The former connection – we now are a Profession, we must have a Subject for ourselves – suggests many inter-connections between the practice and teaching of mathematics, which would be worth exploring.

Since the mid 19th century, historians of mathematics, often trained within the purist regime, have unfortunately reflected these biases in their own historical writings (Grattan-Guinness, to appear). But the author's fragmentary knowledge of meta-history leads him to condemn them to great excess. Apart from various specialist histories of parts of applied mathematics, figures such as Felix Klein (1849-1925) advocated its history in various ways, as part of his furtherance of applied mathematics in general. The flowering of historical work since the mid 1970s has not neglected applications; the steep increase in work on the history of probability and mathematical statistics is especially notable. The central role of applications is the main theme of my recent general history of mathematics cited above, which appeared too late for use by the author; but an 1,800-page encyclopaedia making the same point in much greater detail has been available for some time (Grattan-Guinness 1994).

Understanding the central place of applications in the historical development of mathematics also solves the pseudo-problem posed in Wigner 1960 about "The unreasonable effectiveness of mathematics in the natural sciences". On the contrary, the effectiveness is highly reasonable, even a fine example of rationality, for applications have inspired so much mathematics in the first place. The author notes this paper, but seems to have nothing effective to say about it (at best, p. 41 on invariants).

II

Various links and differences, of both historical and current interest, are also omitted. Firstly, the author seems to take "applications" and "modelling" as synonyms (for example, pp. 11-14). However, in an application some mathematical theory already in existence is deployed, whereas modelling is a more dynamic and inter-active affair in which it may not be clear which mathematics is needed or if it yet exists (Israel 1996).

Secondly, the author restricts "applicable" to "mathematics which is evidently *capable* of being applied *in the future*" (p. 157); but there has also long existed a mass of such mathematics which looks pure in that no application is mentioned (or model cited) but where the background clearly comes from some physical problem. The calculus and mathematical analysis are particularly rich in such material, when it was driven by a differential equation and/or the various functions and integrals arising in its solutions (see Grattan-Guinness 1990, *passim* for many examples between 1750 and 1840).

Sometimes difference equations play the same role, suggesting a third lacuna: the role of numerical mathematics, which has very long and distinguished record (Chabert et al. 1994) and has entered a new golden era of applications now that computers have made feasible many hitherto intractable algorithms. One major consequence has been the growth of non-linear mathematics, to compete with and even replace the linearising hegemony that has lasted for some centuries. In either tradition the expected solution can be fundamentally different: between, say, the calculation of lunar tables to four decimal places and the theoretical analysis of lunar librations. A philosophy of applications based upon the use of hypotheses should surely accommodate considerations of this kind.

III

History sends another warning: that applications can generate mathematical junk. The literature, both ancient and modern, is well stocked with what I call "notional" applications, where the analysis will mention, say, "cooling sphere" or "ring of Saturn" on occasion but in fact is merely an exercise in formula-pushing from which no usable hypotheses about the phenomena allegedly under study can be examined. Numerical methods have their own dangers, such as the "fallacy of misplaced accuracy": the mathematics of, say, $23 \cdot 587498\pm$ about $3\frac{1}{2}$. Modelling can be similarly asinine.

One area of dubious applications and modelling is mathematical economics, which was driven by mechanics and physics throughout its classical and neo-classical phases (Mirowski 1989). As the world of work for mathematicians narrows down to business and finance, one would have expected that this subject and its teaching would be prominent in this book; and one of its most astute commentators, Thorstein Veblen, was a student of Peirce. But nothing is said.

Very tardily, mathematical economics came to be informed by probability and mathematical statistics. These branches of mathematics have been extraordinarily late in their general arrival (Gigerenzer et al. 1989), but now they are of major importance in mathematical education. The status there of hypotheses, using the word in both colloquial and technical senses, forms a cluster of major issues; again it is passed by.

IV

What is the role of Peirce in this apparent revolution? Both more and less than is indicated here.

The main Peircean text is a short passage from a paper now just a century old, in which he stressed the role of hypothesising in mathematics (Peirce 1898, pp. 348-349). But it belongs to a time of increasing talk of hypothesising by scientists, with a gradual change in treating hypotheses seriously as conjectures rather than as candidate certainties (Murphey 1961 *passim*). In isolation it is not exceptional; but Peirce's father Benjamin, professor at Harvard University, is pertinent to it.

A few lines earlier, in a sentence not quoted or discussed by the author, Peirce quoted his father's 1870 definition of "mathematics to be the 'science which draws necessary conclusions' " (Peirce 1898, p. 348). This statement, essential to Peirce's view of hypothesising in mathematics, should have been analysed. Benjamin uttered it at the head of a monograph on associative algebras, written at a time when algebras were proliferating and the utility and especially applicability of many of them was unclear (Grattan-Guinness 1997b). This was a grey area between pure and applied mathematics, exemplifying important issues in the philosophy of algebra which lurk unstudied here.

Part of Benjamin's view was that mathematics does draw the conclusions; the *theory* of drawing them belongs to logic, which his son was developing in an algebraic post-Boolean form. It became the main fixation of Peirce's life, the reddest of the threads running through his life's work (Houser et al. 1996); here it receives one paragraph, uninformative, on p. 175. More than any other logician, Peirce brings to light the perplexing relationship between mathematics and logic, since he saw each as applied to the other. It should attract the interest of a mathematical educator taking Peirce as his starting-point, especially now that the convolutions between logic and mathematics, and also the teaching of both, are being enriched by the steady rise of computer science and use. Again, it is passed over.

The passage is quoted on p. 175, two pages after the author finds it "striking that in Peirce's published work there is almost nothing devoted literally or directly to the subject of applications of mathematics". A slight singularity arises, however; for nearly 20 years Peirce was employed by the United States Coast Survey (for part of the time his father was Director), and in this capacity he published hundreds of pages on the fine details of pendular motion, especially the effects of observations of flexure by the stand (for references and context, see Wolf 1899, 1891). In 1879 he also devised a wonderful map of the world (reproduced in Grattan-Guinness 1997a, p. 606), which deserves to be world-famous.

Among other ignored Peircean details, the author cites only the 1930s Harvard University edition of Peirce's works, a notoriously unsatisfactory source (though sound for the 1898 paper); he seems to be unaware of both the valuable additional two volumes of 1958, and especially of the new chronological 30-volume edition in preparation at Indiana University (Peirce *Writings*). He also omits the fine recent biography (Brent 1995), from which various other points can be learnt.

V

While Peirce the philosopher and scientist is unlucky in the attention given, some other figures gain more of it than might be expected. Imre Lakatos (1922–1974) is praised for "show[ing] that formalistic rigour was a fraud" (p. 170). Sadly, the reference to "Lakatos (1976)" here is ambiguous, since two very different works are so cited in the bibliography (p. 193); but presumably the book version of his discussion of "proofs and refutations" is intended, in which case the characterisation is misleading. The prime aim of his book was to show that theorems and proofs interact, rather than the cause-effect version so often put forward (and *legitimately* studied in formalism, for metamathematical purposes – of which, *pace* p. 17, Cantor's continuum problem is not an example). Lakatos used the nice example of the modifications of Euler's theorem on the vertices, edges and faces of polyhedra, and its later rewritings in histories and textbooks; but no *general* conclusion about mathematical practice, pure or applied, can be drawn from it. A much more varied picture emerges from the writings of Lakatos' mathematical mentor Georg Polya (1887-1985), who is not noted here; see especially Polya 1954 and 1962, 1965.

Lakatos' philosophical mentor was Karl Popper (1902–1994), who is partly blamed by the author for dissuading Lakatos from continuing with the history of mathematics and science (pp. 113-114). In fact, as I can record from personal testimony, Popper would have been delighted if Lakatos had so continued instead of waddling off into a Popperian philosophy of science (the other "Lakatos (1976)" of the bibliography) "that makes nonsense of all my views" (Popper 1974, p. 1000). The author links the alleged discouragement to Popper's low opinion of the philosophy of Wittgenstein; but it is very unlikely that Lakatos would have used that source, or what he might have learnt from it. Wittgenstein inspired a "hard core" (to use a Lakatosian phrase) of admirers, but the bearing of his philosophy upon mathematics education is hard to specify; in particular, his language-games phase was inspired by positivism and its cousin philosophy of behaviourism, both of which mercifully have seen their heyday. In addition, the relationships between philosophers have become very tangled; for, while he did not draw on it substantially, Popper admired Peirce's philosophy and acknowledged anticipation of aspects of his own fallibilism (Popper 1972, pp. 212–216).

Among various other claims, Wittgenstein is credited with inspiring the abandonment from the 1930s onwards of the belief that all words are names (p. 38). However, Russell's very influential 1905 theory of definite descriptions (similar to the criteria for a mathematical function to be single-valued, incidentally) had already begun to instil such disbelief.

VI

The author's critical position against purism is well taken: no wonder that children, and the public at large, detest and even fear mathematics when its professionals extol it as a glass bead game capable of understanding only by the highest of priests. He calls for "polymathematicians" to lead the new approach driven by applications (Chapter 10). No doubt many skills are needed to master not only the mathematics but also the panorama of other disciplines in which it is used or modelled. But the polymathematician needs a much better grasp of his heritage than is visible here, and also a different selection of philosophies from which to be guided: Peirce would be a fine choice, using many other passages. Much of the discussion in this book is either truistic or contentious (especially for those with some historical knowledge), and often long and repetitious. Further, as indicated above, several areas of major concern to its enterprise are omitted. Although heuristics is very much at the core of the author's approach, the work of Georg Polya is ignored.

The most useful parts of the book are the case studies, especially in Chapters 2–3, where various physical problems or tasks are stated usually from modern situations and some taken from technology. For example, "A simulated wood-burning electric fire" is to be analysed using simple algebraic expressions about volumes of wood placed in a hopper and its consumption over time (pp. 61–62). The next case concerns the reading of the letters of the name of a station on a sign by a passenger as a train pulls into a platform – a nice context, but the mathematics involves merely relations of the form ab/c between the parameters (p. 62).

Chapter 4 lists a dozen cases of "Pivotally innovative developmental hypotheses", such as the cost of converting crude oil into fuel or the cost of transplanting human organs; but only common-sense points are made, with no indication of the mathematics to be deployed. What is the educator to do with these examples (p. 69)?

> That it might be possible to put milk in waterproof cartons.
>
> Could a plastic-coated paper carton ever have the durability for this? How often would it leak? How would you prove [*sic*] that it would stand up to the wear-and-tear of ordinary use?

In these and many other cases, the (partial) solutions seem to be too brief or preliminary for the educational benefit or merit to emerge clearly. Seemingly the author has school-level education in mind, though even this is not clearly stated.

Nothing is said either about methods of assessment of work produced under this aegis, although App. F contains several "Peircean modelling scenarios" of practical situations which apparently set as examination questions. Most are modern situations, such as the working of wiper blades on car headlamps; a hypothesis about lighting ancient tombs by means of a sequence of mirrors (p. 179) is obviously notional.

VII

Great care is needed to avoid this praiseworthy emphasis on applications of mathematics degenerating into a new epoch of notionality decorated with uncontrolled "history" and flowery philosophising. The real history and philosophy of applied mathematics and its education is a far richer source of inspiration for those who wish to proceed effectively down the route that the author proposes.

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