Ethnomathematics. Part 2[1]

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Ethnomathematics and Mathematical Education[2]

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Abstract: From the epistemic point of view, mathematics has various aspects and the term ethnomathematics is the most suitable to express this diversity. Ethnomathematics may be defined as:

a) Prototypical activities that take place within a given group and have elements in common such as counting, representing the space, establishing and symbolising relations, reasoning, infering, etc. These activities give the members of the group an insight of the environment in which they live and the ways of interacting with other human beings.
b) A method for interpreting or thinking within a culture and a microculture whose members relate to each other by using a common method of communication. This method is influenced by physical, social and temporal elements that affect and render possible the existence and ability to think of those who share a similar background.

If mathematics is the accepted or protoypical activities of a group of scientists called mathematicians, then ethnomathematics could be defined as a discipline that comprises mathematics. There may be conflicts if a social group rejects the others and assumes the role of authority of mathematical knowledge. However, the debate should lead to the consolidation of a diverse and enriching point of view in a global future where mathematics need to be adapted to the peculiarities of every different culture.

Every social group has implicit ways and explicit methods to acquire a culture. This is also the case in ethnomatematics.

There are many signs in history that reveal a transformation in the enculturation process. This change may be related to politics, the family or society. Although the mathematical enculturation process has not been studied in depth from the ethnomathematical point of view, we think it largely depends on the characteristics of the environment where it takes place. However, some of the aspects of a particular environment may be shared by all cultures.

I have called ethnodidactics the implementation of the different methods of current mathematics enculturation and the study of these methods in the environment where they take place (i.e. official and unofficial mathematics curricula; geographical, political, social and economic elements or conditions; the instructors, their professional attitude and their education, etc.) Ethnodidactics also includes the study of different forms of evaluation, the beliefs about education – or guided enculturation – and its goals. It might be concluded that equity is possible as far as mathematical enculturation is concerned. However, equity depends on the conditions of every different social environment.

Finally, I have established a relation between ethnomathematics and ethnodidactics, by assessment of certain aspects of the professional knowledge acquired by education students. In order to do this I have drawn on an application of L.A. Zadeh's Fuzzy Theory.

Kurzreferat: *Ethnomathematik und mathematische Bildung.* Von einem epistemischen Gesichtspunkt aus gesehen umfaßt Mathematik verschiedene Aspekte und der Begriff Ethnomathematik ist am ehesten dazu geeignet, diese Vielfalt auszudrücken. Ethnomathematik kann wie folgt definiert werden:

a) Prototypische Aktivitäten, die in einer bestimmten Gruppe stattfinden und denen Elemente wie z.B. Zählen, Raumdarstellung, Aufstellen und Symbolisierung von Relationen, Denken, Schließen, usw. gemeinsam sind. Diese Aktivitäten ermöglichen den Gruppenmitgliedern Einsicht in die Umgebung, in der sie leben, und in die Art der Interaktionen mit anderen Menschen.
b) Eine Methode des Interpretierens oder Denkens in einer Kultur oder Mikrokultur, deren Mitglieder durch eine gemeinsame Kommunikationsmethode in Beziehung zueinander stehen. Diese Methode wird durch natürliche, soziale und zeitliche Elemente beeinflußt, die wiederum die Existenz und Fähigkeit derer, die einen gemeinsamen Hintergrund haben, berühren und ermöglichen.

Wenn man unter Mathematik die akzeptierten oder prototypischen Aktivitäten einer Gruppe von Wissenschaftlern, Mathematiker genannt, versteht, dann kann Ethnomathematik als eine Disziplin, die die Mathematik umfaßt, definiert werden. Konflikte kann es dann geben, wenn eine soziale Gruppe die anderen ablehnt und allein eine Autoritätsrolle über die Mathematik beansprucht. Die Debatte sollte jedoch zu einer Konsolidierung eines mannigfaltigen und bereichernden Standpunktes in einer globalen Zukunft führen, in der die Mathematik den Besonderheiten der verschiedenen Kulturen angepaßt werden sollte.

Jede soziale Gruppe hat implizite Wege und explizite Methoden, eine Kultur anzunehmen. Das ist ebenso der Fall in der Ethnomathematik.

Es gibt in der Geschichte viele Anzeichen für eine Veränderung im Enkulturationsprozeß. Dieser Wandel mag mit Politik, Familie oder Gesellschaft in Beziehung stehen. Obwohl der mathematische Enkulturationsprozeß nicht eingehender von einem ethnomathematischen Standpunkt her untersucht worden ist, denken wir, daß er wesentlich von den Merkmalen der Umgebung, in

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welcher er stattfindet, abhängt. Einige Aspekte einer bestimmten Umgebung können jedoch allen Kulturen gemeinsam sein.

Mit Ethnodidaktik habe ich die Implementierung der verschiedenen Methoden der gegenwärtigen mathematischen Enkulturation sowie die Untersuchung dieser Methoden in der Umgebung, in der sie stattfindet (d. h. offizielle und nicht-offizielle Mathematiklehrpläne; geographische, politische, soziale und ökonomische Elemente und Konditionen; Lehrer, deren berufliche Einstellung und Bildung, usw.) bezeichnet. Ethnodidaktik schließt auch die Untersuchung der verschiedenen Evaluationsmethoden mit ein, Beliefs über Erziehung – oder gelenkte Enkulturation – und ihre Ziele. Man könnte schließen, Gerechtigkeit sei, soweit es um mathematische Enkulturation geht, möglich. Gerechtigkeit hängt jedoch auch von den Bedingungen der verschiedenen sozialen Umgebungen ab.

Schließlich habe ich durch Bewertung verschiedener Aspekte des professionellen Wissens von Studenten der Erziehungswissenschaften eine Beziehung zwischen Ethnomathematik und Ethnodidaktik hergestellt. Zu diesem Zweck habe ich eine Anwendung der Fuzzy-Theorie von L. A. Zadeh benutzt.

ZDM-Classification: E20, D20

1. Ethnomathematics. Trying to explain it

Explaining the entire meaning of ethnomathematics is a difficult task for me. Not because of doubts or ignorance, but because of the difficulty that is implied in making an exposition or giving an explanation with words taken from other theories, from a point of view that includes conceptions of the world, of knowledge, of the cultures and the manifestations that can be called mathematics, which altogether make up a different local theory to explain the complete human tasks and its mathematical reflection. This is like a vital beaming, which can be grasped by the eyes, but the necessary words to explain it have not been created yet.

Each trial to explain it entails an increase of comprehension and the designation of a richer and more solid meaning. That is why I will try to search for explanations and words that lead to my approach of what ethnomathematics is and based on authors who share my point of view.

> “‘Ethnosciences’ (Ethnobotany and Ethnoastronomy) are mainly a method of studying the traditional systems of knowledge and cognition. This method uses linguistics analyses to deduce classification systems and categorisation of the natural world of each culture. (...) Ethnomathematics are similar to that regarding. (...) However, in mathematics a deep philosophical debate emerges about the reality of the object of study”. (Ascher 1991, p. 193)

Which external realities or which ideas inside our minds make up the mathematical framework are the questions that several epistemological trends, always under the premises of categories constructed by an Occidental culture and a sole formal science, try to answer. These presuppose some relationships, orders and structures that I consider cannot be generalised without considering each culture because they are representative of the way of thinking and behaving within a culture. Therefore, a contextual relativism defining mathematics is needed, which will lead us to ethnomathematics.

Ethnosciences have a common objective and in the various fields of ethnomathematics the common objective is: “creation and use of abstract patterns. This is the essence of mathematical ideas” (Ascher 1991, p. 194).

But patterns and uses are closely tied to rich and varied cultures which fortunately exist. This is the reason why “mathematical ideas are rich and multifaceted” (Ascher 1991, p. 185) although some constants are accepted by all cultures: “counting and arithmetic, classifying, putting in order ... Ethno embraces: symbols, codes, slangs, myths and also their specific ways of reasoning and inferring” (D´Ambrosio 1985, p. 45).

Ethnomathematics is the answer to the epistemological concerns of a group of scientists, researchers and educators that share a plural conception of mathematical knowledge.

The ethnomathematical assumptions lead to a postmodern relativist movement in the field of mathematics, being a proof that this scientific field has a vivid consciousness that a part of its work object is redefining its ontological position.

Nowadays it is accepted that logic positivism has given place to relativism to explain not knowledge, but types of knowledge in plural. So that the pretended objective of universal justification of knowledge has gained contextualisation, background and verisimilitude. We have passed from “demonstration” objectives to others of “local interpretation”. We have rescued the “expertises” (savoir faire), emerging from different group tasks as “knowledge” and we have given them back their cultural, historical and linguistic roots, which in fact were never lost to them, but were disregarded and not considered valid to explain the reality from the point of view of positivism.

In this scenery, the movement of sociology of knowledge seems to leave apart mathematics. The emergence within the mathematicians and educators community of the epistemological and pluralist worries reveals that though delayed, it follows also the hermeneutic path, participating and sharing the signs of the era of postmodernity.

To show and debate about this mathematics, less universal than what we were “taught”, we have used our words, those of the natural languages of our native homelands. We soon have an idea of the context we are treating. Words are more than sounds and sounds are not universal either in the effects they produce on us when we think of them. Canalised by words, our mathematical knowledge becomes quite peculiar.

To separate the common and the idiosyncratic of the different types of knowledge, classifiable as mathematical, we have many works at our disposal carried out by researchers and teachers (most times both together), qualitative ethnographic techniques, which are as well part of the relativist beliefs of those who use them as tools to collect knowledge manifestations in their natural environment. These techniques are mostly created and adapted from other social research projects, and constitute a part of the contribution to the research project.

Stating these works in order to allow communication and a constructive critique, as well as the comprehension by the information receiver, has become a new objective and we are getting closer to it using both technologies in association with our own expressive means, and we hope that this congress can achieve its objective of facilitating the communication among its participants as well as send the shared information beyond this walls.

Thus, ethnomathematics is the origin of the reflection on mathematics, culture, education and social justice, which is spreading and consolidating by means of the union of the efforts made by sundry people that are following the path first initiated by the International Study Group on Ethnomathematics (ISGEm group).

From its origin in 1985, the international group ISGEm researches problems that involve mathematics and their relationship with society, as one tool of culture and power with influence on the social order.

Their members (Initial Directive Council: Gloria F. Gilmer (President); Gilbert J. Cuevas; Ubiratan D'Ambrosio; Patrick Scott) carry out historical studies, in contexts of interculturality, of deprived social groups, social and political conditions delimiting the mathematics teaching curricula, mathematical knowledge required to carry out the work, mathematics and genre, theoretical-epistemological delimitation of the current mathematical knowledge and its diversity, uses of mathematical knowledge in scientific, technological and social contexts, etc. Their labour can be witnessed in international publications of their members and in the summaries of their Newsletter.

During the last thirteen years the group has become intercontinental. Because of the global geographic repercussion of the studies carried out by the members of this group, of recognised prestige in the United States, Brazil, a large number of countries in Africa, Oceania and South America, and the more recent introduction in Europe and Asia, it has been considered necessary to begin holding meetings of a higher scientific level and maximum international diffusion in order to co-ordinate and get the highest efficacy of the research efforts and actions.

In this sense, the First International Conference on Ethnomathematics (ICEM1) held recently is a historic event which will always represent the group of participants that with great enthusiasm and with no less effort attended the appeal made by Ubiratan D´Ambrosio to organise a scientific debate forum that will periodically meet, and contribute to the consolidation and expansion of ethnomathematics as a way of thinking.

Etnomathematical thought gathers nowadays a hundred researchers from several different areas (anthropology, sociology, philosophy, psychology, art, etc.) and other teachers of mathematics in universities, secondary and primary schools, adults and children teachers, spread all round the world. They all share and work on themes focused on the plurality of the ethnomathematical knowledge and the social and political conditions that influence them. In this group we define *ethnomathematical people* as all those who share a plural vision of mathematics.

In the present, multicultural and intercommunicative world, contextualized mathematics is a cultural product with an undeniable power to resolve problematic situations, with a high capacity for social identification. It is a part of the thought and way to conceive the world. Therefore, a group like ISGEm can become a meeting point for people concerned with mathematical education for development purposes, for the dignity and equity of all the people and for peace.

Ethnomathematics has contributed to the consciousness of epistemological diversity and the doubt about who validates knowledge. That is quite a lot! Now we know that reality is socially built and without objectivity. All scientific constructs emerge from a culture with their inherent characteristics, as does ethnomathematics.

Occidental culture is the only one that separates *wisdom* from *knowledge,* that is, "savoir faire" knowledge or everyday cognition from academic knowledge (backed up by scientific study); to do and to think are not distinguished in other cultures. In European culture the demarcation criterion of knowledge is the pertaining to a scientific field. This criterion is now crumbling.

However it is important not to associate ethnomathematics with Luvien Lévy-Bruhl´s theory of primitive societies whose practices do not constitute knowledge. That is to say, science in its assumptions and these practices constitute a knowledge of a different ontological category and they are called ethnoscience by the aforementioned author and others. At the contrary, ethnomathematics assumes that all cultures have *wisdom* besides *knowledge.* In our work I propose the non-existence of a border or at least that it is *fuzzy,* in the same manner in which knowledge is fuzzy.

In our occidental culture, implicit or inherent culture knowledge is not considered, meanwhile it is not validated as science. But the pragmatic epistemologists think, and so do I, that reciprocally the social validation of scientific knowledge depends on the ethnoscience or practices associated with trades.

2. Ethnodidactics

The ways how multiple mathematics is made are the object of study of "ethnodidactics".

The other attention focus of the ethnomathematical people is education or mathematical enculturation. They focus mainly on the social conditions that make the instruction, the teacher training and the curriculum that is taught, possible. Within our mathematical perspective, I have called all this cosmos of education "ethnodidactics" (Oliveras 1995b).

It is a main interest focus because it constitutes the enculturation practices of future generations which perpetuate the mathematical manifestations of cultures. They are of undeniable interest not only to those involved in the professional world of education, but also to parents and citizens with children who are minors, adults with an aim of improving within their professions, politicians, thinkers, mass communication, ecologists and publicists, all sorts of products sellers, the designers of leisure spaces-time and domination strategists. We all wonder how it is possible to penetrate inside the lives of groups of people and participate in their referents and their conducts, values and emotions. This achievement is what we call to educate, teach-learn, enculturate a social group that already owns this universal knowledge or any partial manifestation, mathematics in our case.

On one hand ethnodidactics embraces teachers, pupils and the genuine relationships existing among them, which are also important. On the other hand it analyses the fundamentals that underlie the formal educative systems of each country or autonomous group and which affect mathematical education, as well as the analyses of the social framework that lies under the informal and hidden enculturation, independent from educational systems.

In this sense it is clear that there is more than one type of enculturation. In our basic system MEDIPSA (Oliveras 1995b), it is demonstrated how certain approaches or conceptions of what *m*athematics is (M), knowledge (E, of *e*pistemology), organisation of education (D, of *d*idactics), the influence of the researches and their methods (I, of *i*nvestigation), *p*sychological and *s*ociological visions of the learning-teaching process (P and S), in the characterisation of what happens while learning or enculturating with the *a*nthropological cosmovision (A) of human groups and the influence that the diverse current agents of enculturation exert on them, will lead to different systems of education in a wide sense and of mathematical enculturation.

Schools convey what is experienced in life; the values that are used in school regarding coexistence and what is needed to survive in one's environment. What is intentionally taught at school does not always go beyond the school hours and the class walls.

The means by which diverse cultures and microcultures enculturate themselves make possible the perceiving of how societies use "multimathematics" while generating it at the same time.

The study of the generating means of mathematics, of enculturating children and youth is one of the vastest fields of contributions. A great deal of education professionals have begun to observe multimathematics as the solution to the contradictions that have come up and have made them change their point of view about the universal and generic mathematics that everybody has to learn.

We obtain some of the elements of ethnodidactics by taking the above-mentioned studies and extracting experienced forms of action and contexts from them in which evidence is found of the generation of mathematical knowledge, which in some facets is characterised by ethnomathematics.

Starting from these elements we can generate new interpretative studies that show the local conditions that make mathematical knowledge possible without establishing cause and effect relationships but metaphorical analogical models, which are objects of reflection and the way to prepare new educative performances.

The belief that mathematical knowledge is transmitted by the asymmetric fund of knowledge that a text or a teacher has accumulated is only correct when the pupil has already gained interpretation means which allow him/her to decode the message. The following question is: How has he/she achieved this capacity? How has he/she learned to listen comprehensively? This concrete didactics of oral transmission is only one of the possibilities and maybe the less efficient at present.

In our didactic relativism we will need to describe each enculturation process and look for the effects it has caused, those expected and those unpredictable, within the macro processes in which each young person trying to learn gets involved and which constitute new contributions of knowledge elements coming from outside of the intentional educative world, which altogether represents the introduction of mathematical thought in his/her cultural experiences.

When somebody has an ethnoknowledge it means that an ethnodidactic process has taken place by which this knowledge has been integrated in his/her cosmos. We can capture this process by means of anthropo-sociological techniques and we are obtaining ethnodidactic elements, of the functioning didactics, permitting the production of mathematical knowledge.

For instance, the Euro as the single European currency issues a challenge to combine child mental structures, supposedly additives, with an ordinary multiplicative situation which leads to real decimal numbers until the disappearance of the peseta (and the remaining current currencies) and until we think constantly in Euros. The school cannot leave apart the problem until the child reaches the age of 12. It will be necessary to explain the situation to all the children whatever their age. How will this be achieved?

When this moment arrives the monetary referent in Spain will be the Euro (the provisional exchange calculated in May 1998 is equivalent to 168 "pesetas") an amount much higher than the current "peseta" and the relativity of the economical concepts will produce an increase of the marginalized group for economic reasons. The concepts of rich and poor, as relative as all the other concepts, are the less analysed from an anthropological and equity approach in school mathematical education. It is alarming how the concepts of rich and poor are part of the current social situation that makes the set of values teeter. There are no African immigrants in Europe, because if they are rich they are called "investors", but if they are poor, or they are looking for a job illegally, they are a problem.

Some parts of ethnodidactics are discovered with great difficulty by means of very expensive ethnographies; "expensive" due to the human and financial means required. The use of curricula and their previous intentional definitions; school practices and their artefacts; hidden curricula inherent to values assumed by teachers, managing and administrative agents which participate in the school environment. All of this and the social aspects regarding cohabitation and civic attitudes, assessment of the teacher's mathematics task, his/her own concept as a professional, the sort of human and scientific relationship kept with the students as an educational variable, the grouping together of these professionals considering their critique or conformist visions of the official work situation and the relationship between leadership and the values that are taught. These subjects are all object of study and research in ethnodidactics.

Therefore, ethnomathematics and ethnodidactics represent what multimathematics is and how it is socially generated in the current world, in which we have achieved the philosophical maturity reflected in the abandonment of the concerns for the essentials and the worries for groups of sentient beings which live with integrity and without which nothing makes sense.

3. Diffuse modelling of ethnodidactic elements

Antonio Machado was the author of the aphorism "Walker, there is no path; it is made upon walking." Machado was a genial Spanish poet, thinker and teacher who died in exile because he doubted the apparently firm and sure dominant knowledge and power.

I have taken his words in this moment to support my option for uncertainty and relativistic dialogue, as the way to achieve a certain level of truth regarding mathematics and the processes that involve learning it, which we have called ethnodidactics.

Lakatos proclaimed a long time ago the loss of certainty as regards to mathematical knowledge. Removing the foundations of science that has been considered as the maximum landmark of human abstraction and simultaneously was considered a language instead of science ... Wittgenstein was the first in identifying science and language, but afterwards he doubted the generality of both and created the basis for a science concept that rejects concepts, general truths. The reason for this apparently non sequitur is explained by Wittgenstein, by an example (of course!), referring to games.

Our ethnomathematics presents multiple examples or ways to produce mathematics as an explanation for the plurality and contextualization of this science. It is a science that makes use of many languages to express itself. Thus, we can achieve more certainty than we expected after the mathematical foundations crisis.

The formal mathematical language is one among various which is utilised in ethnomathematics, and which I have used to express certain ethnodidactic elements relating to the acquisition of professional knowledge by teachers in their training process. I carried out a practical research project with these future teachers based on ethnomathematics.

The students of teacher training, in order to enculturate themselves didactically by the acquisition of knowledge characteristic of their work as teachers, collaborated with me in the preactive and active phases of development of some curricular topics in their activities which we elaborated from an ethnomathematical approach. These activities were organised under a curricular product which I called Microprojects (Oliveras 1995a, b).

A group whose aim was research in practice was formed. By the constitution of this group called Algabar (Oliveras/Grupo Algabar 1996), they became aware of their incipient professional identity. The research was guided by me, who was as well their university professor, and in this way they could establish a very close and strong relationship with the culture they wanted to learn. The links were not the conventional ones that exist between teacher and student, but ones generated by a symmetric and active relationship in which the professor, researcher and student roles are interchanged and produce knowledge in the environment.

To characterise and assess some of this knowledge, I think it is appropriate to use the recently created language of fuzzy mathematics, on which Zadeh has theorised.

The theory of fuzzy sets appeared in 1965 with the publication of the article *Fuzzy Sets* by L. A. Zadeh. It intends to give mathematical solutions to vague terms of language and to the necessity for having a mathematical representation of "real life" sets, where sometimes definitions do not have clear borders. It has diverse applications for the evaluation of not well-defined processes. For this reason, A. Jones has used it in the evaluation of human acquisitions and imprecise qualities, as for instance, creativity.

This mathematics is suitable to measure components of the professional knowledge of teachers, since, as Alsina and Trillas (1991) say: "Maybe it is time to consider fuzzy sets and fuzzy logic as powerful tools for mathematically modelling some interesting facts that integrate aspects of human knowledge". The type of facts the previous authors are referring to are not only cognitive but also educative.

The training of teachers of mathematics presents some imprecise features, that can be modelled by means of fuzzy logic and so we can resolve some of the problems that come up in this line of didactics of mathematics. In our study we use A. Jones' technique of fuzzy evaluation (Jones et al. 1986) to assess the ethnodidactic professional mathematical knowledge of teachers in training, thus obtaining a new application within the education field. It can be witnessed in Espin and Oliveras (1997).

Fuzzy logic may be considered as a multi-valued logic in which an element has a greater degree of set membership than simply the fact of "belonging to" or "not belonging to" and which supports types of approximate reasoning instead of exact ones.

The notion of an element's membership of a fuzzy subset is determined by a membership function that determines the degree to which the said element belongs to a subset.

> A fuzzy subset A is defined by means of the membership function, which assigns to each element x a real value from the interval $[0, 1]$. The membership function (μ_A) expresses the degree to which x verifies the characteristic of A:
>
> $$A = \{x \in U \mid (x, \mu_A(x))\}$$
>
> $U =$ universal set of the discourse, or reference set.
>
> According to Zadeh, approximate reasoning is the process of obtaining consequences by starting out from imprecise statements. The consequence of this reasoning may be imprecise or vague, depending on the degree of compliance of the background elements.

A. Jones applied a fuzzy model to the field of education and several theoretical constructs relating to assessment, amongst which was a technique for assessing the deviation of a student's knowledge with respect to the knowledge taken as a reference, which may be the teacher's knowledge or a fuzzy defined knowledge (Jones et al. 1986).

By applying the technique described above to our case, we can make an assessment of the fuzzy deviation of the trainee teachers' professional, ethnodidactical-mathematical knowledge, taking the lecturer as a reference and as text questions, their system of emergent didactic categories that refer to conditions of infant learning in the activities that we have called microprojects. These categories have to be taken as a fixed reference, that is to say, that the assessment measurement of the reference is always right. Therefore, the lecturer's fuzzy measurement is always equal to one, thus we shall achieve:

$$\mu_M(c_i) = \{(c_1, 1),\ (c_2, 1), (c_3, 1),\ (c_4, 1),\ (c_5, 1),\ (c_6, 1), (c_7, 1), (c_8, 1)\}.$$

Since, in the context of the microproject studied there are eight categories.

In order to determine the level, we take a scale of values included between 0 and 1:

$$0,\ 0.25,\ 0.50,\ 0.75 \text{ and } 1.$$

These values shall correspond, in our case, to the degree of knowledge that a student-teacher possesses about each one of the categories in the test set.

The values are awarded on the basis of the reports prepared by the students on their teaching tasks during the period of teaching practice. These reports were studied and using a technique for analyzing the teaching content, categories were drawn up. For awarding the values, we divided the range of each total interval by establishing four equal intervals therein. The students were coded with a number and a letter.

Thus, for student 3T, we have:

$$\mu_{3\mathrm{T}}(c_i) = \{(c_1, 1),\ (c_2,\ 0.75),\ (c_3,\ 0.75),\ (c_4,\ 0.5),\ (c_5,\ 0.25),\ (c_6,\ 0),\ (c_7,\ 0.5), (c_8,\ 0.5)\}.$$

Assessment of the student's knowledge, in accordance with the fuzzy deviation defined by Jones:

$$\{|\mu_M(c_i) - \mu_S(c_i)|\} = \{d_S(c_i)\},$$

S = student, M = lecturer,

as a fuzzy subset associated with each student, is defined by taking a fuzzy subset from the set of values of the distances of the pupil with respect to the teacher in each reference item. In our case, we shall obtain the fuzzy deviation, with respect to the lecturer, of the ethnodidactical-mathematical capacity of the seven trainee teachers, whose case we studied, and which is represented by the following n-tuples (each letter of the student indicates a different microproject and so a different number of categories that corresponds to a different size of the n-tuple):

Teacher training	Didactic-mathematical capacity measured and deviation
μ(3E)= desv(3E)=	(1, 1, 0.5, 0, 0.25, 0.5, 0.5, 0.25) (0, 0, 0.5, 1, 0.75, 0.5, 0.5, 0,75)
μ(5E)= desv(53E)=	(1, 0.5, 0.5, 0.5, 0, 0.75, 0.25, 0) (0, 0.5, 0.5, 0.5, 1, 0.25, 0.75, 1)
μ(14T)= desv(14T)=	(1, 1, 0, 0.5, 0.5, 0.5, 0.5, 0.25, 0) (0, 0, 1, 0.5, 0.5, 0.5, 0.5, 0.75, 1)
μ(26T)= desv(26T)=	(1, 0.5, 0, 0.5, 0, 0.25, 0.25, 0.25, 0.5) (0, 0.5, 1, 0.5, 1, 0.75, 0.75, 0.75, 0.5)
μ(31T)= desv(31T)=	(1, 1, 0, 0.5, 0.25, 0, 0.25, 0.25, 0.25) (0, 0, 1, 0.5, 0.75, 1, 0.75, 0.75, 0.75)
μ(1A)= desv(1A)=	(1, 1, 1, 0.75, 1, 0.5) (0, 0, 0, 0.25, 0, 0.5)
μ(2A)= desv(2A)=	(1, 1, 1, 0.5, 1, 0.5) (0, 0, 0, 0.5, 0, 0.5)

This assessment by reference to the teacher provides us with the ideal pupil in relation to that teacher as the one with nil deviation in all his components, or elements in the fuzzy subset that defines the deviation.

We define the following relationships between pupils and between pupils and the teacher, in partial order: a pupil or teacher "k" relates to the other "m", if "k" shows less deviation than the latter, "m", with respect to the teacher in the same category, in the same context or microproject.

It is obvious that the teacher is related to them all.

A pupil-teacher proximity relationship may be obtained by means of a graph showing the type of fuzzy statements that may be made about the capacity in the pupils and between them.

In our case, the proximity relationship for two pupils is given on the following table:

Categories	Pupil deviation (A1)	Pupil deviation (A2)	Teacher deviation
1ª	0	0	0
2ª	0	0	0
3ª	0	0	0
4ª	0.25	0.5	0
5ª	0	0	0
6ª	0.5	0.5	0
N of relationships	1	0	12 (6 with 1A and 2A in every category)

In this relationship of partial order, pupil 1A has a relationship of greater proximity to the teacher than pupil 2A and this corresponds to category 4A. In the five remaining categories they are equidistant.

It may be noticed that there are four categories in which they coincide with the teacher reference, and only in categories 4 and 6 there is no coincidence, this equals 66% of the possibilities. This percentage and its source mean that these two cases are the best situated ones in the assessment. Case 1A is the most ideal one.

The study above (Espin/Oliveras 1997), shows us that fuzzy logic may be used to measure and assess trainee teachers' professional ethnodidactical knowledge, regarding microprojects as curricular products and regarding the categories emerging that we provide in the original research work (Oliveras 1995b) as indicators that will allow fuzzy comparative assessment of the students (Jones et al. 1986).

This assessment is, overall, positive, since the results are suitable for the levels for which they were proposed.

The results of the research allow the different aspects of teaching to be collected and interpreted, such as:

- Difficulties of the teacher's professional work
- Capacity to organize mathematical learning
- Expression of concepts and learning achieved
- Acceptance of mathematical activities outside the school environment, which constitute contextualised ethnodidactic mathematical knowledge, achieved by the trainee teachers who were enculturated through the project above mentioned, whose ethomathematical approach gave the elements needed to initiate a change in

the epistemological conceptions of the future teachers.

4. References

Alsina, C.; Trillas, E. (1991): Fuzzy Sets and Mathematics Education. – In: For the Learning of Mathematics 11(3), p. 16–19
Ascher, M. (1991): Ethnomathematics. A Multicultural View of Mathematical Ideas. - New York: Chapman & Hall
D´Ambrosio, U. (1985): Ethnomathemathics and its Place in the History and Pedagogy of Mathematics. – In: For the Learning of Mathematics 5(1), p. 44–48
D´Ambrosio, U. (1996): EducaÇao Matemática. Da teoría à prática. – São Paulo: Papirus
D´Ambrosio, U. (1998): Mathemathics and peace: Our responsibilities. – In: ZDM Zentralblatt für Didaktik der Mathematik 30(3), p. 67–73
Espín, A.; Oliveras, M. L. (1997): Introduction to the use of fuzzy logic in the assessment of mathematics teachers´ professional knowledge. – In: Mathematics – Education & Applications. MC.MEA-97. Cyprus Mathematical Society, p. 107–113
Frankenstein, M.; Powell, A. (Eds.) (1997): Ethnomathematics. Challenging Eurocentrism in Mathematics Education. – Albany: S.U.N.Y. Publ.
Jones, A.; Kaufmann, A.; Zimmermann, H. J. (1986): Fuzzy Sets. Theory and Applications. – In: Series C: Mathematical and Physical Sciences Vol. 177, p. 349–395. Dordrecht: Reidel
Oliveras, M. L. (1995a): Artesanía Andaluza y Matemáticas. Un trabajo transversal con futuros profesores. – In: Revista UNO II(No. 6), p. 73–84 (Barcelona, ISSN 1133-9853)
Oliveras, M. L. (1995b): Etnomatemáticas, formación de Profesores e innovación curricular. – Granada: Comares, 1996. Parts of the Doctoral Thesis (ISBN 84-8151-241-9)
Oliveras C., M. L. (1996): Panorama de la investigación en Etnomatemáticas. Una investigación sobre artesanías en Andalucía. – In: Cultura y Matemáticas, p. 39–60 (ISBN:84-7801-341-5)
Oliveras C., M. L. (1997a): Reflexiones sobre el perfil del profesor de Matemáticas del próximo siglo. Alternativa para la Formación de Profesores de Matemáticas. – In: Publicaciones Universidad de Concepción, Seminario Internacional sobre Formación de Profesores, p. 49–74. Santiago de Chile (ISBN 956-272-797)
Oliveras C., M. L. (1997b): Etnomatemáticas y educación intercultural. – In: Educación. ¿Integración o exclusión de la diversidad cultural? p. 161–170 (ISBN 84-921390-3-x)
Oliveras C., M. L.; Grupo Algabar (1996): Memoria colectiva de autodescripción del Grupo Algabar. Investigación en el aula de Matemáticas. – In: Investigación en el aula de Matemáticas: El currículo, p. 245–267 (ISBN 84-920554-5-6)
Zadeh, L. A. (1992): Knowledge Representation in Fuzzy Logic. An Introduction to Fuzzy Logic Applications in Intelligent Systems. Chapter 1. – Dordrecht: Kluwer Academic Publ.

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Allgemeine Mathematik: Mathematik und Realität

Darmstadt, 29.9. – 1.10.1999

Mit der Tagung ”Allgemeine Mathematik: Mathematik und Realität” wird eine Tagungsreihe fortgeführt, die mit den Tagungen ”Allgemeine Mathematik – Mathematik für die Allgemeinheit” (1995), ”Allgemeine Mathematik: Ordnen, Strukturieren, Mathematisieren” (1996), ”Allgemeine Mathematik: Mathematik und Bildung” (1997) und ”Allgemeine Mathematik: Mathematik und Lebenswelt” (1998) begonnen wurde. Die Tagungen sollen dazu beitragen, eine breite Auseinandersetzung über Mathematik und ihre Bedeutung für die Allgemeinheit zu fördern; dabei soll es vor allem um Reflexion des Selbstverständnisses der Mathematik, ihres Verhältnisses zur Welt sowie um Fragen nach Sinn und Bedeutung mathematischen Tuns gehen. In diesem Rahmen ist auch das Thema ”Mathematik und Realität” zu verstehen. Auf der kommenden Tagung sollen deshalb Fragen diskutiert werden wie:

- Welchen Realitätsstatus hat die Mathematik?
- Wie wird Mathematik in der realen Welt wirksam?
- Was ist die gesellschaftliche Bedeutung der Mathematik?
- Wie wird die Entwicklung der Mathematik durch reale Probleme beeinflusst?

Wie schon die vorangehenden Tagungen soll auch die ’99er Tagung Wissenschaftler/-innen und wissenschaftlich Interessierte aus unterschiedlichen Bereichen wie vor allem der Mathematik, Didaktik, Erziehungswissenschaft, Philosophie und Informatik zusammenführen.

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