

## LEVELS OF STUDENT RESPONSES IN A SPREADSHEET-BASED ENVIRONMENT

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*The purpose of this report is to investigate the range of student responses in three domains - hypothesizing, organizing data, and algebraic generalization of patterns during their work on a spreadsheet-based activity. In a wider context, we attempted to investigate students' utilization schemes of spreadsheets in their learning of introductory algebra. Twenty students' responses to an investigative assignment were analyzed. The findings indicate a wide range of student responses. In each of the three domains analyzed, most student responses fell into several clearly definable categories. However, an attempt to establish a hierarchy of performance levels led to less clear results.*

### Background

Researchers and educators suggest using various models of learning environments, that widen and enrich the scope of learning processes for students having differing mathematical abilities. Technological tools were recognized as a particularly effective means to achieve this purpose (see for example, Balacheff & Kaput, 1996).

The *Compu-Math* learning project created a technologically based learning environment, which systematically covers the entire mathematics syllabus for grades 7-9. As described by Hershkowitz and her colleagues (2002), this project is based on the following principles:

- Investigation of open problem situations;
- Work in small heterogeneous groups, where the problem is investigated and discussed;
- Consolidation of the mathematical concepts and processes, that arose in group work, in a whole-class discussion;
- Investigations that utilize computerized tools to facilitate operations within and between various mathematical representations, to reduce the load of formal algorithmic work, to enable the construction of mathematical concepts and processes, to provide feedback to hypotheses and solution strategies, and to resolve a real need to explain important processes and products;
- Interactions between students in a group or in the class as a whole, between students and computerized tools, and between students and the teacher;
- Reflective actions on the learning process;

This report investigates the range of student responses in three domains: (1) hypothesizing, (2) organizing data, and (3) algebraic generalization of patterns during students' work in a spreadsheet based activity. The findings address the broader issue of detecting the learning processes underlying introductory algebra students' *instrumental genesis* (Mariotti, 2002). Here we attempted to describe and discuss the processes involving students' perception and utilization of spreadsheets as a technological artifact.

Our choice of domains was guided by their importance in the process of learning mathematics in general, and in the context of spreadsheet-based learning of introductory algebra in particular, and also by their potential to characterize and possibly indicate levels of mathematical learning. The importance of each of the three investigated domains was considered by others on various occasions:

- Chazan and Houde (1989) and Howe and his colleagues (2000) indicate that students' hypothesizing has the potential to be a source of authentic mathematical activity, a catalyst for meta-cognitive action, and a motivator for learning.
- Student performance at the stage of collection and organization of data was investigated in particular in the domain of data analysis (Ben Zvi & Garfield, in press) and spreadsheet-based mathematical activities.
- Algebraic generalization of patterns is considered central for the learning of algebra on the one hand, and as a source of student cognitive difficulties on the other hand (Kieran, 1992).

## **Methodology**

One of the authors (M. T.) was the teacher-researcher of a Grade 7 introductory algebra class consisting of 24 students. For this particular group, all five weekly lessons were conducted in a computer laboratory; students had access to a computer, and occasionally used *Excel* spreadsheets as tools for mathematical work and documentation. In Grades 5 and 6, the students were also occasionally engaged in exploratory mathematical activities that employed *Excel* spreadsheets. During selected lessons, the teacher conducted both video and audio recordings. The video recordings included the class work of one pair of students and discussions involving the entire class. The audiotapes recorded the discussions of 3-4 additional pairs of students. In addition, all the students were required to save their computer work on the net. The data analyzed in this report is based on the data collected during one 90-minute lesson. The data include transcriptions of the video recording of one pair, the audio recordings of four pairs, and 12 spreadsheet files of all students. Twenty students attended this lesson, and worked in 6 pairs, one group of three, and 5 individuals. The lesson analyzed here was based on an activity called *Growing Rectangles*, and was conducted at an early stage of the course – three weeks after the beginning of the school year.

## The activity of *Growing Rectangles*

The situation associated with the problem (Figure 1) presents the process of growth of three rectangles; students are requested to relate to the first ten years of this process.

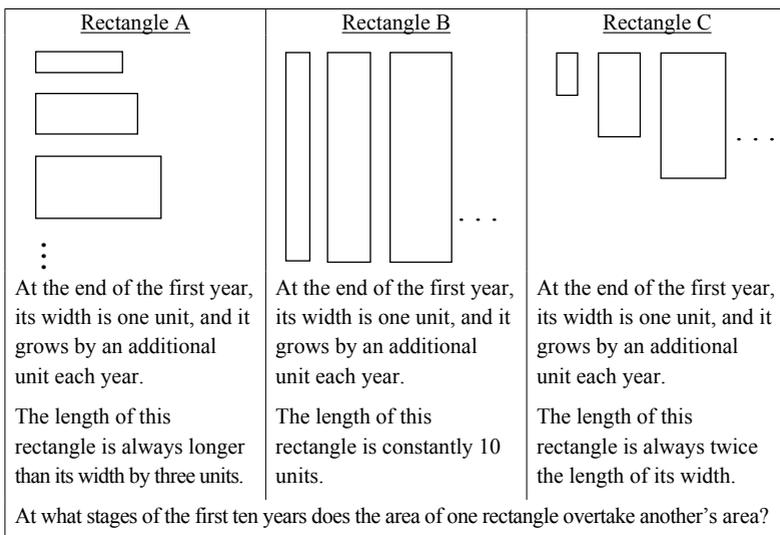


Figure 1. Problem situation of the *Growing Rectangles*.

At the initial stage, the students are required to predict (hypothesize), without performing any calculations or formal mathematical operations, which rectangle will overtake another's area, and at what stage.

Next, the students are required to organize their data regarding the growing rectangles in a spreadsheet table, record the formulas used to construct their table, and compare (first numerically and then graphically) their findings and their predictions.

### Student responses

Next, we will attempt to present and analyze some categories of student responses to this activity. As previously mentioned, we will restrict our analysis here to three domains: hypothesizing, organizing data, and algebraic generalization of patterns.

**Hypothesizing (predicting results).** In this domain, we observed three categories of responses.

- *Local considerations.* Most students sampled one or more points on the time sequence and drew conclusions according to their findings in these selected points. For example, two pairs chose to look at the fifth year (probably because of its being the midpoint of the given period of ten years) and realized that at this point, the areas of Rectangles B and C are equal. This led them to conclude that these

rectangles become equal in area every five years. They rejected this hypothesis at a later stage.

- *Considerations of rate.* A few students considered the rectangles' rate of growth. One pair reasoned as follows: Rectangle B has a fixed length, and as a result, it cannot "win". Between Rectangles A and C, Rectangle C is the "winner", since at each step "it grows by itself".
- *Eliminating the common variable.* One pair noticed that one side of each rectangle is the same at each stage and grows similarly. As a result, they ignored the contribution of this side to the area, and considered only the growth process of the other side. The comparison of the three corresponding sequences of lengths (Rectangle A - 4, 5, 6, ...; Rectangle B - 10, 10, 10, ...; Rectangle C - 2, 4, 6, ...) led them to conclude that Rectangle C will have the largest area at the end of the process.

Ben Zvi and Arcavi (2001) found that student analyses that are based exclusively on local considerations lead to poorer results, as compared to an argumentation that is based on global, or combined global and local considerations. Thus, we considered the first category of hypothesizing to be at a lower level, in comparison to the other two. We could not develop an argument with regard to the comparison of the other two categories. They are both global and based on general features of a variation process. One should note, however, the elegant simplicity of the third strategy.

**Organization of data.** In this activity, the students were required to use *Excel* in order to collect, organize, and analyze their data. However, they were not instructed *how* to organize their data. Figure 2 presents the four categories of tables observed in the students' work files and their corresponding frequencies.

- *Separate tables.* Students in this category constructed three tables - one table for each rectangle (Figure 2a). Each table contained four columns to describe the year (from 1 to 10), the rectangle's two linear dimensions, and its corresponding area.
- *Extended table* (Figure 2b). The tables of this category contained ten columns (i.e. variables): the year, six columns for the linear dimensions of each rectangle, and three columns for the area measures. The columns were in order either by rectangle (i.e. an annexation of the three separate tables previously described) or by variable (i.e. first, grouping together the linear dimensions of all rectangles and then, their area measures).
- *Reduced table* (Figure 2c). These students allotted only one column for the width measures, since they are identical for all three rectangles. Thus, the number of columns in these tables was reduced to eight.
- *Minimal table* (Figure 2d). These students noticed that the width measures are identical to the year number and omitted the width measures altogether. Moreover, they omitted the length measures as well, and included their corresponding expressions directly into the area formulas of each rectangle. Thus, the number of columns in this case was further reduced to four.

Rect. A	Width	Length	Area
1	1	4	4
2	2	5	10
3	3	6	18
4	4	7	28
5	5	8	40
6	6	9	54
7	7	10	70
8	8	11	88
9	9	12	108
10	10	13	130

Rect. B	Width	Length	Area
1	1	10	10
2	2	10	20
3	3	10	30
4	4	10	40
5	5	10	50
6	6	10	60
7	7	10	70
8	8	10	80
9	9	10	90
10	10	10	100

Rect. C	Width	Length	Area
1	1	2	2
2	2	4	8
3	3	6	18
4	4	8	32
5	5	10	50
6	6	12	72
7	7	14	98
8	8	16	128
9	9	18	162
10	10	20	200

(a) Separate tables (2 files)

Year	Width Rect. A	Length Rect. A	Area Rect. A	Width Rect. B	Length Rect. B	Area Rect. B	Width Rect. C	Length Rect. C	Area Rect. C
1	1	4	4	1	10	10	1	2	2
2	2	5	10	2	10	20	2	4	8
3	3	6	18	3	10	30	3	6	18
4	4	7	28	4	10	40	4	8	32
5	5	8	40	5	10	50	5	10	50
6	6	9	54	6	10	60	6	12	72
7	7	10	70	7	10	70	7	14	98
8	8	11	88	8	10	80	8	16	128
9	9	12	108	9	10	90	9	18	162
10	10	13	130	10	10	100	10	20	200

(b) Extended table (4 files)

Year	Width	Length Rect. A	Area Rect. A	Length Rect. B	Area Rect. B	Length Rect. C	Area Rect. C
1	1	4	4	10	10	2	2
2	2	5	10	10	20	4	8
3	3	6	18	10	30	6	18
4	4	7	28	10	40	8	32
5	5	8	40	10	50	10	50
6	6	9	54	10	60	12	72
7	7	10	70	10	70	14	98
8	8	11	88	10	80	16	128
9	9	12	108	10	90	18	162
10	10	13	130	10	100	20	200

(c) Reduced table  
(2 files)

Year	Area Rect. A	Area Rect. B	Area Rect. C
1	4	10	2
2	10	20	8
3	18	30	18
4	28	40	32
5	40	50	50
6	54	60	72
7	70	70	98
8	88	80	128
9	108	90	162
10	130	100	200

(d) Minimal table (4 files)

Figure 2. Categories (and frequencies) of tables observed in student *Excel* work files.

First, a distinction can be made between the construction of separate tables and the other categories. Students who employed the first strategy did not consider the common features of the three rectangles and the task at hand. A numerical or graphical comparison of several processes of variation requires either a common table or a common graph. An analysis of the other three table categories led us to conclude that an increasing level of conciseness is related to higher level of reasoning. As indicated by the findings presented in the next section, the construction of a compact table is related to the abilities to detect patterns and to express symbolically the relationships involved in this particular problem situation.

**Algebraic generalization of patterns.** Hershkowitz and her colleagues (2002) indicate that the use of spreadsheets to investigate processes of variation enables students to use spontaneously algebraic expressions. Spreadsheet users employ formulas (expressed in spreadsheet syntax) as a natural means to construct extensive numerical tables and then, possibly to plot graphs. In our case, after three weeks of learning algebra, all students, with one exception, were able to write and then copy (“drag”) spreadsheet formulas, to obtain the necessary numerical data. We investigated whether the formulas used by the students in this case have the potential to indicate levels of student ability to generalize algebraically. After examining students’ work in this activity, we formulated the following categories:

- One student *exclusively used numbers* and showed no attempt to generalize, but was still able to construct a graph based on his numerical data.
- *Recursive formulas* express a relationship between two consecutive numbers in a sequence. Figure 3(b) presents an example of using recursive expressions for obtaining the dimensions of a rectangle.
- *Explicit formulas* use the sequence place index as an independent variable. In our case, 3 students (in 3 files) used the year number as an independent variable in their expressions (see Figure 3(a)).

	A	B	C	D
1	Year	Width	Length	Area
2	1	=A2	=A2+3	=A2*(A2+3)

(a) Explicit formulas.

	A	B	C	D
1	Year	Width	Length	Area
2	1	4	1	=A2*B2
3	=A2+1	=B2+1	=C2+1	=A3*B3

(b) Mixed recursive and multivariate formulas.

	A	B	C	D
1	Year	Width	Length	Area
2	1	=A2	=A2+3	=B2*C2

(c) Mixed explicit and multivariate formulas.

Figure 3. Algebraic generalizations.

- *Multivariate formulas* use more than one variable to express a generality. In our case, in 8 (out of 12) files the area of the rectangles was expressed by using the letters corresponding to the length, width or year columns (e.g., = B2\*C2). The

variables used in a multivariate formula were originally obtained by a recursive method or by an explicit formula (see Figures 3(b) and (c)).

Recursive formulas can be considered to be the result of a local view of a pattern. In standard algebra, recursive formulas are less effective as a tool for finding a required number in a sequence, or for analyzing and justifying sequence properties. In a spreadsheet environment, these disadvantages are less valid and hence less obvious to students (or researchers). The spreadsheets' dragging ability allows us to obtain a very large quantity of numbers by using *any* kind of formula – including a recursive or a multivariate one. Moreover, the same action of dragging enables students to understand the global aspects of a recursive formula. Recursive formulas have a didactical advantage as well. For example, they are easier to understand and produce, and sometimes their use is the only way that some complex (for example, exponential) functions can be introduced at an early stage.

Multivariate formulas are also frequently considered an obstacle to students' performance in algebra. Lee (1996) states that one of the main difficulties in algebraic modeling is not the construction of a general expression, but the finding of a model that proves to be effective in the solution process. Once again, the difficulty of producing an ineffective model is bypassed by the spreadsheets' ability to accept and handle a considerably wider range of generalizations than with a paper-and-pencil environment. In a spreadsheet environment, students frequently replace a quantity previously expressed as an algebraic expression by a new variable. Jensen and Wagner (1981) consider students' ability to view expressions as entities a characteristic of algebraic expert thinking. The contribution of this strategy to advance this skill needs further inquiry.

### Summary

Our analysis of student responses in this spreadsheet activity revealed a wide range of student responses. Because of the variety of student responses detected in our findings, we concluded that a spreadsheet-based learning environment enables students to follow different paths of *instrumental genesis*, according to their algebraic reasoning and their perception of the employed artifact.

In addition, we attempted to create categories of responses with regard to students' ability to hypothesize, to organize data and to generalize. In each of these three domains, most student responses could be categorized in several distinctive groups. However, an attempt to distinguish levels of performance among these categories led us to less clear results. In our case, the process of hypothesizing did not require the employed technological tool. As a result, we established levels of performance by an analysis of student mathematical reasoning.

The activity presented here required students to organize large quantities of numerical data. Spreadsheets are particularly well-suited to facilitate the construction of tables. Our findings indicate that this feature enables students of all

levels to organize their data. Moreover, we distinguished various levels of performance in this domain, based on students' level of mathematical understanding of the task.

With regard to students' algebraic generalizations, we found that the spreadsheets' powerful mathematical capabilities enable students to obtain the required results by employing strategies that are considered ineffective in a paper-and-pencil environment. As a result, we could not establish a hierarchy of generalization skills that would be valid for both environments. We also recommend that the effect of work with spreadsheets on students' ability to generalize algebraically in both environments be investigated.

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