

THE SENSUAL AND THE CONCEPTUAL: ARTEFACT-MEDIATED KINESTHETIC ACTIONS AND SEMIOTIC ACTIVITY

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In this paper we deal with the problem of the genetic relationship between the sensual and the conceptual in knowledge formation. Transcript and videotape analyses of two small groups of a regular Grade 11 mathematics class shed some light on the dialectics between semiotic activity and artefact-mediated kinesthetic actions. The analyses point to a dialectical embedding of the sensual and the conceptual through perceptual activity, gestures, mediated action, speech, and signs.

INTRODUCTION

Although 20th century psychology acknowledged the role of language and kinesthetic activity in knowledge formation, and even though elementary mathematical concepts were seen as being bound to them (as in Piaget's influential epistemology), bodily movement, the use of artefacts, and linguistic activity, in contrast, were not seen as *direct* sources of abstract and complex mathematical conceptualizations.

Nevertheless, recent research has stressed the decisive and prominent role of bodily actions, gestures, language and the use of technological artefacts in students' elaboration of elementary, as well as abstract mathematical knowledge (Arzarello and Robutti 2001, Robutti 2003, Nemirovsky 2003, Núñez 2000). In this context, there are a number of important research questions that must be addressed. One of them relates to our understanding of the relationship between body movement and actions carried out through artefacts (objects, technological tools, etc.) with linguistic and symbolic activity. Research on the relationship between these two chief sources of knowledge formation (i.e. artefact-mediated kinesthetic actions and semiotic activity) is of vital importance for a better understanding of human cognition in general, and of mathematical thinking in particular. As current research suggests, highly complex mathematical symbolism cannot incorporate students' kinesthetic experience in a direct manner. The severe limitations of a direct translation of actions into symbols require the students to undergo a dynamic process of imagining, interpreting and reinterpreting.

The goal of this paper is to contribute to our understanding of the dialectical process between (concrete or imagined) actions, signs and meanings underpinning students' elaboration of mathematical conceptualizations. In order to do so, we continue and deepen our analysis of a classroom activity where Grade 11 students were asked to investigate the relationship between the distance traveled and the time spent by a cylinder moving up and down an inclined plane (Radford et al. 2003). In this paper,

we focus on the students' difficulties in interpreting a graph involving negative distances.

THEORETICAL FRAMEWORK

In our theoretical framework, the interplay between semiosis and artefact-mediated kinesthetic actions is located in the individual's reflective, cognitive activity, out of which certain conceptual objects are grasped or produced. The semiotic-cultural perspective that we are advocating stresses the fact that the production of knowledge of individuals results from goal-oriented activities, encompassed by social processes of meaning making. In the course of such processes, stable forms of awareness are achieved and subjective intentions are made apparent. In this context, awareness and intentions as subjective constructs are consubstantial with the variety of culturally embedded actions carried out through a dynamic interaction of semiotic systems (e.g. gestures, speech, written language, mathematical symbolism), perceptual activity, and tool use. These culturally embedded actions underlining the individual's elaboration of mathematical knowledge are part of complex social processes of *knowledge objectification* (Radford 2002, 2003).

Knowledge objectification nevertheless requires the possibility of detaching oneself from the particularities of personal experiential perspectives. As Piaget remarked, actions on objects provide the individual with a vantage viewpoint on the object of knowledge, a vantage viewpoint that, ontogenetically, has later to undergo a process of detachment (or "*décentration*", to use Piaget's word). The same is true of perceptual activity and of natural language too. With its arsenal of deictics, (e.g. "I" "here", "there", "now") natural language indeed anchors the individual's talk at a particular spatial-temporal point. It is from this point that reference to objects of discourse is made. This point accounts for distinctions between e.g. "here" and "there". Its spatial-temporal nature becomes apparent when we notice that if we move this point, "right" may become "left", "close" may become "far", and so on.

As a result of the contextual nature of actions and of the aspectual view deriving from language, gesture and perceptual activity, a spatial-temporal relationship is created between the individual and the conceptual object leading to what can be termed an *embodied meaning*. This embodied meaning has to become somehow *disembodied* in order to endow the scientific conceptual object with its cultural, interpersonal value.

Within the sketched semiotic-cultural framework, the relationship between semiotic activity and artefact-mediated kinesthetic actions can be seen as a dialectical relationship in which external cultural conceptual objects are transformed into objects of self-awareness through an integrated movement unfolding between the worlds of the sensual and the conceptual (i.e. between the manifold of sensual experience and the world of conceptual idealities). To better understand this dialectic, in this paper we discuss some steps in the students' process of knowledge objectification as knowledge becomes objectified in the physical, cultural environment of body and artefacts –an environment that we propose to view as a semiotic system, i.e. as a system of signs and significations. We shall return to this point later.

METHODOLOGY

Data Collection: Our experimental data comes from an ongoing longitudinal classroom-based research program whose classroom activities (elaborated by a team of teachers, researchers and research assistants) are part of the regular school teaching lessons, as framed by the provincial Curriculum of Mathematics. In these activities, designed as layered *zones of proximal development* (Vygotsky), the students spend a substantial part of the activity working together in small groups of 3 or 4. At some points, the teacher conducts a general discussion allowing the students to expose, confront and discuss their different solutions. In addition to collecting written material, tests and activity sheets, we have three or four video-cameras each filming one group of students. Subsequently, transcriptions of the video-tapes are produced. These transcriptions allow us to identify salient short passages that are then analyzed in terms of the students' use of semiotic resources and tool use.

The Teaching Sequence: The data reported here comes from the second day of a two-day mathematical activity based on a hands-on investigation of motion along an inclined plane. The first day, the students were asked to make a graph of the relationship between the time spent and the distance traveled by a cylinder propelled from the bottom of a ramp. Then the students carried out the experiment using a TI 83+ calculator connected to a Calculator Based Ranger (CBR) placed on top of the ramp. The students were asked to compare their graph to the one produced by the calculator and to discuss the differences between the two. One of the questions asked to the students on the second day was the following: "A group of students drew the following curve to represent the relationship between time and space when a cylinder is propelled upwards [from the bottom] on an inclined plane [see Fig. 1]. This group placed the distance origin around the center of the inclined plane. Is this curve correct? Explain in detail your answer." Transcript and videotape analysis of one of the small groups suggested that some students' difficulties are linked to the distinction between the mathematical origin and the origin of the cylinder motion (Radford et al. 2003, pp. 60-61). In this paper, we will focus exclusively on this question. We want to deepen our analysis of this cognitive problem in order to shed some light on the more general problem of the dialectics between semiotic activity and artefact-mediated kinesthetic actions underpinning the students' processes of knowledge objectification.

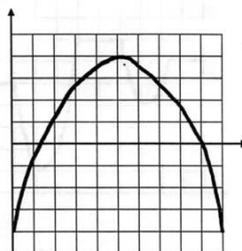


Figure 1. The graph accompanying the question.

RESULTS AND DISCUSSION

The classroom mathematical activity required the students to coordinate two different semiotic systems: on one hand, the semiotic system of body and artefacts where a concrete experiment was performed, on the other, the Cartesian coordinate semiotic system. Each of them is governed by its own semiotic structure. In the first case, the semiotic structure derives from the particulars of the experiment, e.g. the position of

the CBR, the place where the cylinder was propelled, the moment in which the cylinder started moving, the place where the cylinder was stopped, etc. (see Figure 2)



Figure 2. The picture shows the physical environment of body and artefacts as a semiotic system of signs and significations. It includes the ramp, the cylinder, and noteworthy points such as the CBR position, the beginning and the end of the cylinder motion, body position, etc. The cylinder was propelled upwards from the bottom of the ramp. The picture captures a moment of the cylinder motion after release.

In the second case, the semiotic structure obeys mathematical conventions (e.g. a division of the domain along two perpendicular axes, one for registering distances from their origin and the other for registering the elapsed time). The mathematical origin of the Cartesian system is of course an important point in the sense that the relationship between variables is referred to it. The contextual nature of the experiment endows a structure to the physical semiotic system of body and artefacts, where other significant points can be identified. They are defined by their mutual relationship and by the role they play in the actual course of the experiment. One of the most noteworthy points is the reference point of the students' spatial-temporal mathematical experience –the point from where an embodied meaning is bestowed on signs. Following Bühler's linguistic concept (Bühler, 1979), we want to term this point the *origo*.

In what follows we present excerpts from two small groups (these groups belong to the same classroom of the group mentioned in Radford et al. 2003). As will be seen in the course of the analysis which follows, the first group had problems distinguishing between the mathematical origin (0,0) of the Cartesian coordinate system and the *origo*. The second group, in contrast, managed to establish a suitable distinction. In asking the students to critically judge whether a given graph was correct or not, knowing that the mathematical origin corresponded to a point near the middle of the ramp, negative distances come into play offering us an interesting terrain in which to investigate our research question, that is, the dialectics between the sensual and the conceptual.

In the following excerpt the students were studying the graph shown in Figure 1.

1. Tammy: I think it doesn't make any sense ... you can't just go down to the negatives like that ... (*she point with her pen to the lower part of the graph*)
2. Amanda: Why would it go down into the negatives? (*Pointing to the ramp, which is on the right of the students, she says*) it goes down into the negatives ... if the ball falls (*i.e. if the cylinder goes off the ramp*) (see Table 1 Picture 1 from left to right).

3. Jess: Unless they threw it up from the floor (*as she utters the word up she makes a gesture with her right hand moving it up, see Picture 2*) and it fell backwards.
4. Amanda : Ok well we can just say that for us it doesn't make any sense because when you begin to throw the ball your minimum point is at zero (*with her right hand she makes a gesture indicating a point on her desk; see Picture 3*) and it can't be in the negatives, it's at zero (*she moves now the hand back to mean below the origin; see Picture 4*) [...] the minimum point of the ramp is zero [...] zero signifies the end of the ramp.
5. Jess : ... if anything, it should be like this [...] it would be straight here and straight there (*she adds two segments to the graph where it cuts the horizontal axis; see Picture 5*).



Table 1. Pictures 1 to 4 (from left to right) show parts of the dialectical process between imagined actions, meaning and the interpretation of the sign-graph (shown in picture 5), as encompassed by physical experiments carried out on the first day.

In these lines, the students display an attempt at making sense of the given graph. Tammy starts making a general statement. In line 2 Amanda elaborates further and offers a first explanation that links the conceptual category of negative numbers and the physicality of the experiment in a decisive way. As her utterance reveals, the conceptual realm remains subjected to the sensual experience by the metaphor that the cylinder “goes down” into the negatives. This metaphor rests on two key elements:

- (1) The physical aspect of the phenomenon where one actually *sees* the cylinder *moving down* in the last part of its trajectory (a part that is stressed by Amanda's gesture shown in Picture 1), and
- (2) The central idea that the bottom of the ramp is taken as “zero”.

Within this conceptual space, the only way negative values could be obtained, as Amanda says, is if the cylinder goes off the ramp. The confusion between the physical “zero” (i.e. the *origo*, defined by the spatial coincidence of body and cylinder motion) and the mathematical origin impedes a suitable understanding. In keeping with this view, Jess suggests the unlikely possibility that the cylinder was thrown up from the floor. Pictures 3 and 4 in Table 1 show Amanda producing a gesture (a “moving down” gesture) that simulates the imagined motion of the cylinder going below the bottom of the ramp. The “moving down” gesture serves the purpose of knowledge objectification. An alternative is proposed by Jess in Line 5, where the graph is emended in a coherent way within the students' working conceptual space: no part of the graph can go below the horizontal axis, so little horizontal segments are added in each part where the graph cuts the axis.

As we see, the students did not succeed in suitably objectifying knowledge. Knowledge remained confined to the embodiment of mediated actions. Kinesthetic,

mediated actions were governing both the realm of sensual experience and of mathematical conceptual descriptions, leading to a superimposition of the mathematical origin and the *origo*.

Let us now turn to the second group of students. In the beginning, this group also had problems relating negative numbers to the cylinder motion:

1. Sandra : But a value can't be negative, it can't be negative ... minus 2 meters.
2. Nelly : Negative 2 meters, even if she walks backward it can't be negative [...]
3. Sandra : Ok, (*she proposes an imaginary situation*) Albert goes for a walk and one measures his distance.
4. Albert : So I go then I come back, the instant that you move, it isn't minus meters.
5. Nelly : Yeah ... If you walk one meter behind you'll say, yoh man! I walked a negative mile!
6. Sandra : Or if you take a ball and you dig a hole in the ground, the ground has a value of zero ... whatever.
7. Nelly : Still the ball wasn't in the negative meter ... it went down ...
8. Albert : But it will come back to 2 meters, even though ... I don't know...

In this part, in an attempt to make sense of the graph, the students had recourse to two experiential situations: a walk and the position of a ball, each of them referring to their own *origo*. The inclusion of the conceptual category of negative numbers in the students' discussion about practical (imagined) situations shows the interplay between the sensual and the conceptual. But again, the predominance of the physical *origo* over the conceptual mathematical origin impedes the coordination between semiotic systems. After some more discussions, the students continue as follows:

9. Sandra : Let's say that this is zero (*she points to a point on the ramp; see Table 2, Picture 1*), we'll say that this is zero ...
10. Nelly : (*Interrupting*) there it goes (*she makes an indexical gesture with her right hand that physically touches a point on the ramp. Then she takes the cylinder with her left hand and puts it right beside the point indicated by her index finger on the ramp and says*) but do they start counting here?
11. Sandra : (*Disagreeing with the fact that Nelly put the cylinder beside her index finger, says*) Yeah but they begin to count ...
12. Nelly : **If** they count that as point zero! Then it's like ... (*she moves the cylinder to the bottom of the ramp; see Picture 2*)
13. Sandra : (*Interrupting*) Well yeah it's negative variable...
14. Nelly : (*At this moment, Nelly starts a long gesture: she moves slowly her left hand while saying*) So negative one,... zero (*she stops for a very short moment; see Picture 3*)
15. Albert : (*When Nelly reaches the point of zero, he says, at the same time as Nelly*) Zero...
16. Nelly (*Continuing to move the cylinder up says*), one, two (*she reaches the maximum point –see Picture 4*) and when moving the cylinder back to the bottom (*says*) blah, blah, blah ... (*see Picture 5; then, turning to her group-mates, says*) I guess that could go

In this excerpt, Sandra starts working on a hypothetical situation derived from the problem data, namely, that the distance origin was placed around the center of the

surface where the cylinder can move. She makes an indexical gesture (Table 2, Picture 1) that is elaborated in a more precise way by a second gesture performed by Nelly. Indeed, Nelly turns to the ramp (which was behind her) and positions her index finger to indicate in a precise way the point on the ramp that corresponds to the mathematical zero of the distance axis. Thus far, things are not yet completely clear, as is shown by her putting the cylinder close to her right index. Next, Sandra makes her understand that the cylinder did not start there. As she is still not fully convinced, Nelly uses the conditional “if,” which she stresses by a clear intonation. Within the hypothetical situation thus defined, with her left hand she places the cylinder on the bottom of the ramp, which corresponds to the physical origin or *origo*. In so doing, the *origo* and the zero of the distance axis are perceptually distinguished in an unequivocal manner. It still remains to be seen if there is agreement between the two semiotic systems –the one of body and artefacts and the Cartesian graph. The enactment of the cylinder motion follows. The coordination between the two semiotic systems is accomplished by perceptual activity and by another semiotic system: *speech*. While she moves the cylinder up, she says “negative one” “zero” “one”, “two”. A suitable understanding seems to have been reached, and the second part of the cylinder motion is now merely schematized with gestures and words. Indeed, the words “blah, blah, blah ...” now indicate a few points on the ramp that do not require further specification.



Table 2. Indexical gestures indicate the point on the concrete semiotic system of body and artefacts that corresponds to the mathematical origin of the distance variable. The enactment of the cylinder motion, encompassed by mediated actions and conceptual categories, leads to the objectification of knowledge.

But knowledge objectification is not something that usually happens once and for all. Thus, the students’ dialogue continues with a reflection on the meaning of zero. Replying to Nelly’s last utterance (Line 16), Sandra says:

17. Sandra : Since the origin isn’t at zero-zero I guess it could go...
18. Albert : (*Looking at Sandra he says with a subtle and Machiavellian smile*) What is zero-zero...?
19. Sandra : Mmm ... Well ... that would be like the bottom of the ramp (*she points to the bottom of the ramp with her pen*).
20. Albert : (*interrupting*) Zero-zero is there where you put it ... when we had the CBR (*referring to the experiments carried out the day before*) we had zero at the CBR, so it depends on where you want to put the origin ... so technically it’s true (*Sandra and Nelly think for a while about what Albert has just said*) (...)
21. Sandra : But really because it’s from the bottom ... but there isn’t really a CBR so we don’t know (...)
22. Albert : It’s where you want to put the origin... it makes sense! It’s where you want to put the origin!

While on Line 8 Albert was still unsure about the difference between the origin and the *origo*, we now see that the difference has become well established. Through a complex interplay of conceptual categories, gestures, perceptual activity, and mediated imagined and concrete actions, the dialectics between the sensual and the conceptual ensured a culturally correct interpretation of the graph.

CONCLUDING REMARKS:

As the previous analysis suggested, one important difference between the two groups of students mentioned in this paper is the possibility to accomplish a detachment of body and actions. Such a detachment is underpinned by the disembodiment of a meaning generated by artefact-mediated kinesthetic actions. The disembodiment of meaning is not related to the exclusion of the body or the austerity of the action. Rather, the disembodiment of meaning is related to the possibility of dialectically embedding the sensual and the conceptual. This dialectic embedding is most revealing in the “semiotic node” (Radford et al. 2003) of perceptual activity, gesture, mediated action and speech displayed in lines 4 to 8 (see also Table 2). Nelly’s indexical gesture indicating the origin of the distance variable opens up the possibility to re-cognize the domain of scrutiny where the iconic gesture comes to enact the motion of the cylinder. The iconic gesture is accompanied by words that highlight spots to be perceptually attended, leading to the attainment of a stable form of awareness. Gestures, mediated actions and words mark altogether, out of the continuum of the ramp, particular points that are simultaneously sensual and conceptual. Naturally, the short examples discussed in this paper do not exhaust the difficult problem of the epistemic relationship between the sensual and the conceptual. Our examples merely suggest that our understanding of students’ objectifying processes of historically and culturally constituted bodies of knowledge require a better understanding of the dialectics in which the sensual and the conceptual become subsumed into each other. They point to some of the aspects of the general problem and thereby call for further theoretical and practical research.

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