

MATHEMATICAL MODELLING WITH YOUNG CHILDREN

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This paper addresses the first year of a three-year, longitudinal study which introduces mathematical modeling to young children and provides professional development for their teachers. Four classes of third-graders (8 years of age) and their teachers participated in the first year of the program, which involved several preliminary modeling experiences followed by two comprehensive modeling problems over a span of 6 months. Regular teacher meetings involving preparatory workshops as well as reflective analyses were conducted. Analyses of children's responses (in group situations) to one of the modeling problems shows the spontaneous ways in which children engaged in sense making, problem posing, hypothesizing, and mathematizing (including representing). It is argued that modeling tasks of the present type are powerful vehicles for developing important mathematical ideas and problem-solving processes in the early school years.

INTRODUCTION

Children today are facing a world that is shaped by increasingly complex, dynamic, and powerful systems of information in a knowledge-based economy (e.g., sophisticated buying, leasing, and loan plans that appear regularly in the media). Being able to interpret and work with complex systems involves important mathematical processes that are under-emphasized in numerous mathematics curricula, such as constructing, explaining, justifying, predicting, conjecturing and representing, together with quantifying, coordinating, and organising data. Dealing with such systems also requires students to be able to work collaboratively on multi-component projects in which planning, monitoring, assessing, and communicating results are essential to success (English, 2002; Lesh & Doerr, 2003). The primary school is the educational environment where all children should begin a meaningful development of these processes and skills (Jones, Langrall, Thornton, & Nisbet, 2002). However, as Jones et al. note, even the major periods of reform and enlightenment in primary mathematics do not seem to have given most children access to the deep ideas and key processes that lead to success beyond school.

In the study addressed here, we introduced 8-year-olds to model-eliciting activities that focus on structural characteristics of phenomena (e.g. patterns, interactions, and relationships among elements) rather than surface features (e.g. biological, physical or artistic attributes). Our longitudinal study addresses both the children's and teachers' growth across three years; however, for this paper we consider the development of the children's mathematization processes as they worked their first modeling activity. One of our initial goals was to introduce the children (and their teachers) to mathematical modeling. To this end, we developed and implemented a number of preliminary modeling activities, which we describe later.

MATHEMATICAL MODELLING FOR YOUNG LEARNERS

Traditionally, students are not introduced to mathematical modeling until they reach secondary school (Stillman, 1998). However, the rudiments of mathematical modeling can and should begin much earlier than this, where young children already have the foundational competencies on which modelling can be developed (Diezmann, Watters, & English, 2002; Lehrer & Schauble, 2003; NCTM, 2000). Young children's problem-solving experiences have traditionally been limited to problems in which children apply a known procedure or follow a clearly defined pathway. When it comes to solving these problems, the "givens," the goal, and the "legal" solution steps are usually specified unambiguously—that is, they can be interpreted in one and only one way. This means that the interpretation process for the child has been minimalised or eliminated. The difficulty for the child is simply working out how to get from the given state to the goal state. While not denying the importance of these existing problem experiences, they do not address adequately the mathematical knowledge, processes, representational fluency, and social skills that our children need for the 21st century (English, 2002; Steen, 2001). An important component of problem solving in today's world is interpreting the problem situation, dealing with ambiguous or incomplete information, identifying constraints on solutions, and visualising and evaluating possible end-products.

Mathematical modeling takes children beyond basic problem solving where meaning must be made from symbolically described word problems, to authentic situations that need to be interpreted and described in mathematical ways (Lesh, 2001). At the same time, these modeling activities encourage multiple solution approaches and call for multifaceted products. Another important feature of these activities is that key mathematical constructs are embedded within the problem context and are elicited by the children as they work the problem. Children not only have to work out how to reach the goal state but also have to interpret the goal itself as well as all of the given information, some of which might be displayed in representational form (e.g., tables of data). Each of these aspects might be incomplete, ambiguous, or undefined; furthermore, there might be too much or too little data, and visual representations might be difficult to interpret (as in real-world situations). When presented with information of this nature, children might make unwarranted assumptions or might impose inappropriate constraints on the products they are to develop.

Modeling activities for children are also social experiences (Zawojewski, Lesh, & English, 2002) and are specifically designed for small-group work, where children are required to develop explicitly sharable products that involve descriptions, explanations, justifications, and mathematical representations. Numerous questions, conjectures, conflicts, revisions, and resolutions arise as children develop, assess, and prepare to communicate their products. Because the products are to be shared with and used by others, they must hold up under the scrutiny of the team members.

RESEARCH DESIGN AND APPROACH

The study is using multilevel collaboration, which employs the structure of the multitiered teaching experiments of Lesh and Kelly (2000), and incorporates Simon's (2000) case study approach to teacher development. Among the features of multilevel collaboration is a focus on the developing knowledge of all participants, who work as co-investigators operating at different levels of learning (English, 2003). At the first level of learning, children work on sets of modelling activities, which engage them in constructing, refining, and applying mathematical models. At the second and third levels respectively, research students and classroom teachers work collaboratively with the researchers in preparing and implementing the child activities. At the fourth level, the researchers observe, interpret, and document the knowledge development of all participants. Multilevel collaboration is most suitable for this study, as it caters for complex learning environments undergoing change, where the mechanisms of development and the interactions among entities are of primary interest (Salomon, Perkins, & Globerson, 1991).

Participants: Four third-grade classes (8 years old) and their teachers participated in the first year of the study. The classes represented the entire cohort of third graders in a school situated in a middle-class suburb of Brisbane, Australia. The school principal and assistant principal provided strong support for the project's implementation. They were informed of the progress of the study, and attended some of the workshops and debriefing meetings that we conducted with the teachers.

Procedures and activities: We conducted two half-day workshops with the four participating teachers in term 1 to introduce them to the modelling experiences and to plan more thoroughly the year's program. Two more workshops were conducted during the middle and at the end of the year for planning and reflective analysis of the children's and teachers' progress. Several shorter meetings were also conducted throughout the year, including those before and after the teachers had implemented each activity. The contexts of the modelling activities were designed to fit in with the teachers' classroom themes, which included a study of food, animals, and flight.

Preliminary modelling activities were implemented by the teachers towards the end of first term and part of second term. These activities were designed to develop children's skills in: (a) interpreting mathematical and scientific information presented in text and diagrammatic form; (b) reading simple tables of data; (c) collecting, analysing, and representing data; (d) preparing written reports from data analysis; (e) working collaboratively in group situations, and (f) sharing end products with class peers by means of verbal and written reports. For example, one activity involving the study of animals required the students to read written text on "The Lifestyle of our Bilby," which included tables of data displaying the size, tail length, and weight of the two types of Bilbies. Children answered questions about the text and the tables. In another activity focussing on food, the children read about the development of

chocolate from the growth of the cocoa bean to the manufacture of various types of chocolates. Tables of data on the ingredients found in various chocolate types were included. After answering a number of questions on the given information, the children implemented their own survey about chocolate consumption, gathered and analysed the data, and then reported their findings to their peers.

During the remainder of second term and for all of third term, the teachers implemented, on a weekly basis, the main model-eliciting activities. Each lesson was of 40 minutes duration. After an initial whole class introduction, the children worked independently in groups of 3 to 4 to complete the activity. The first model-eliciting activity, which we address here, was titled “Farmer Sprout.” It comprised a story about the various types of beans the farmer grew, along with data about various conditions for their growth. After responding to questions about the text, the children were presented with the “Butter Beans” problem. Here, the children had to examine two tables of data displaying the weight of butter beans after 6, 8, and 10 weeks of growth under two conditions (sunlight and shade; see Table 1).

Table 1. Data presented for the Beans Problem (Students were advised that the farmer had grown 4 rows of butter beans under two light conditions)

Sunlight			
Butter Bean Plants	Week 6	Week 8	Week 10
Row 1	9 kg	12 kg	13 kg
Row 2	8 kg	11 kg	14 kg
Row 3	9 kg	14 kg	18 kg
Row 4	10 kg	11 kg	17 kg

Shade			
Butter Bean Plants	Week 6	Week 8	Week 10
Row 1	5 kg	9 kg	15 kg
Row 2	5 kg	8 kg	14 kg
Row 3	6 kg	9 kg	12 kg
Row 4	6 kg	10 kg	13 kg

Using the above data, the children had to (a) determine which of the conditions was better for growing butter beans to produce the greatest crop. In a letter to Farmer Sprout, the children were to outline their recommendation and explain how they arrived at their decision; (b) predict the weight of butter beans produced on week 12 for each type of condition. The children were to explain how they made their prediction so that the farmer could use their method for other similar situations. On completion of the activity, each group reported back to the class.

All whole class interactions were videotaped, while the group work was videotaped (one group per classroom) and audiotaped (2 groups per classroom). We also took field notes of all the class and group activities. The teacher meetings were audiotaped and transcribed, along with the class and group activities.

SOME FINDINGS: CHILDREN'S RESPONSES TO THE BEANS PROBLEM

Prior to addressing some findings, it is worth commenting that each of the teachers established classroom norms for group work and for class reporting. For example, each group of children had a group-appointed manager who was responsible for organising materials and keeping the group on task. The importance of sharing ideas as well as explaining answers was also emphasised. First we consider some general observations of the children's progress on the Butter Beans Problem and then examine the work of one group of three children.

In general, we noted an initial tendency for the children to want to record an answer from the outset, without carefully examining and discussing the problem and its data. The children had to be reminded to think about the given information and share ideas on the problem prior to recording a response. We also observed the children oscillating between analysing the data and discussing at length the conditions required for growing beans. At times, the children became bogged down in these practical discussions, with their progress slowing as a result. This point is revisited shortly.

We noted at least three approaches that the children adopted in analysing the data in Table 1. The first approach was to focus solely on the results for week 10 and systematically compare rows 1 to 4 for each condition (i.e., compare 13 kg with 15 kg, 14 kg with 14 kg and so on). A variation of this approach was to make the comparisons for each of weeks 6 and 8 as well. A second approach was to add up the amounts for week 10 in each condition and compare the results. A third but inappropriate variation of the last approach was to sum all of the weights in each table and compare the results. As one child explained, "Sunlight has 146 to 118 (shade). So plants are in sunlight." A further approach (again, inappropriate) was to add the amounts in each row for each condition and compare the end results (i.e., 9kg + 12kg + 13kg for sunlight and 5kg + 9kg + 15kg for shade, and so on).

As the children explored the data, they were looking for trends or patterns that would help them make a decision on the more suitable condition. They were puzzled by the anomalies they found, and resorted to their informal knowledge to account for this:

Students collectively: 10 against 6, 11 against 10, and 17 against 13.

Student 1: So this is obviously better than that, but working out why is the problem.

Student 2: Yes, because the more sunlight the better the beans are. For some reason...

Student 1: In some cases, it's less; but in most cases, it's more the same.

Student 3: It would depend on what type of dirt it has been planted in.

Student 2: I've got an idea. Perhaps there were more beans in the sunlight.

Student 3: We're forgetting one thing. Rain. How much rain!

The above group of students spent quite some time trying to suggest reasons for the trends in data and became waylaid by practical issues. Nevertheless, in doing so, the children engaged in considerable hypothetical reasoning and problem posing, for

example: “I’ve got an idea. If we didn’t have any rain, the sunlight wouldn’t ...it wouldn’t add up to 17 (kg). And, if we didn’t have any sunlight, it wouldn’t be up to 17 either. But if we had sunlight and rain....”

For the remainder of this lesson, the above group of students cycled through finding practical reasons for *why* they thought sunlight was better, re-examining the sets of data, and attempting to record their findings. The children did not make substantial progress during this lesson largely because of their focus on finding reasons for the trends in the data. As one child explained to the researchers, “But our problem is, we thought it would be because of the rain. It can’t get in as well with the shade cloth on. But then we found these results. And we’ve got a problem. We can’t work out why this has popped up. So we’re stuck here.”

Considerable progress was made in the second lesson, however, after one of the children (Amy) directed the others to focus solely on the data. The child directed their attention to row 3, week 10, where the difference was the greatest (“Here’s the best and here’s the worst”). Amy attempted to show the other group members this difference by drawing a representation of the two amounts. Her representation took the form of a simple bar graph (“picture graph”, as she described it) with the first bar coloured yellow to represent the 18kg (sunlight) and the second bar coloured black to represent the 12kg (shade). In directing her peers’ attention to her diagram, Amy explained, “O.K., so that would be about the sunlight there.....what I am trying to say is the shade is about half as good as sunlight.” Her peers, however, were not paying attention so she decided to pose this question to bring them back on task: “This here is sunlight and this here is shade. Which one’s better?” Still not happy with her peers’ lack of enthusiasm, Amy posed a more advanced question for her peers:

Amy: Oscar, if this long piece was shade, and the short piece was sunlight, and they represented the weight of the beans, which one would be better?

Oscar: This.

Amy: No, shade would be because it’s bigger. A bigger mass of kilograms.

The difficulty for many of the children was completing the letter for Farmer Sprout. As Amy explained to the teacher, “You see, I’ve drawn a picture graph and we’ve worked out the answer, but we can’t put it into words... I know! We can draw this (her representation) on our letter and explain what it means in words. And that’ll get us out of it.” The group finally produced the following letter, choosing to focus solely on the largest difference between the conditions:

Dear Farmer Sprout, We have decided sunlight is the best place to grow Butter Beans. Because if this was the best (*an arrow pointing to the representation*) and this the worst (*another arrow pointing to the representation*), black = shade and yellow = sun. 18kg and 12kg. It is obvious that sunlight is better because 18 is higher than 12 by six. We came to this decision because sunlight maenley (sic) projuded (sic) more kg or the amount of kgs. Yours sincerely, Mars Bars (*name of the group*).

In reporting their findings to the class, the above group commented that “Shade produced about half as much as sunlight altogether.” When asked where they obtained their information for this conclusion, Amy explained, “Well, we basically

added all of this up (week 10 data for each condition) and we found that shade produced about half as much as sunlight altogether.”

In responding to the second component of the Butter Beans Problem, the children generally relied on patterns in the data to predict the mass of the beans after 12 weeks. For example, another group in Amy’s class reported their predictions for the sunlight condition as follows: “Our findings show that in row 1, week 12, you will get 15 to 17 kilograms, and in week 12, row 2, you’ll get 17 kilograms, and in row 3, week 12, you will get 19 to 21 kilograms, and in week 12, row 4, you shall get 18 to 20 kilograms. That’s what we think for sunlight.” When asked how they got these findings, the children explained, “The data, because we went to week 10 and we counted 2 on....because they’ve sort of gone up like, in twos and it was another two.” It is interesting to note that in one class discussion on the likely mass of the beans at week 12, some children again extrapolated beyond the data and claimed that at week 12 the beans would be too old and would probably die.

CONCLUDING POINTS

The model-eliciting activities used in our study encourage young children to express ideas related to structural characteristics of meaningful phenomenon. We saw varying levels of sophistication in the mathematization processes of the children. There appeared to be several elements that either facilitate or constrain the growth of these processes. First is the ability to interpret and understand data presented in various representational formats. Although the majority of children had few difficulties here, we observed a few children who misinterpreted what was being measured (i.e., they thought that the number of kilograms referred to the amount of sunlight rather than the mass of beans). A second element was an intuitive knowledge and understanding of the mathematical concepts inherent in the task, such as the notions of change and rate of change. Intuitive notions of aggregating and averaging were expressed by some of the student groups. A third element was the issue of existing personal knowledge versus task knowledge. Being able to distinguish between the two knowledge forms, and knowing how they can both help and hinder solution facilitate task success. We see the development of such metacognitive and critical reasoning skills as important components of these modelling activities. Fourth, the modelling tasks provided rich opportunities for children to express ideas in multiple representations. We saw how children used text, diagrams, and verbal explanations to engage in effective learning in a socially mediated environment.

There appear to be several implications for teachers. First children who engage in social learning contexts verbalise their thinking and hence make explicit their knowledge of content and processes. Opportunities thus exist for teachers to extend children’s knowledge in areas of need. At least one of the participating teachers often capitalised on discussions to make links to historical information that the children may have previously learned. Some links were made to scientific or social studies but few explicit connections were made to mathematics. The modelling activities were embedded in a broader theme being explored by the class but explicit links were

often not taken up by the teachers who appeared to assume that the activity needed to be isolated for the benefit of the researchers. Opportunities for children to actually implement the activity described in the modelling task can help young children who might lack some of the related conceptual knowledge. In the present case, undertaking the task of comparing growth conditions of plants (either in real life or via computer simulation) can provide concrete representations that facilitate model development.

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