LEARNING TO USE CAS: VOICES FROM A CLASSROOM

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This paper reports on the experiences of students who were learning mathematics with CAS for a second consecutive school year. Evidence presented shows that nearly all students managed the challenging task of mastering the technical aspects of using CAS well. It also shows that the level of technical difficulty and the degree to which it presents an obstacle to mathematical learning is not predictable from conventional mathematical ability. There is a complex interaction between cognitive and affective factors. Planning appropriate teaching for developing the effective use of CAS will require awareness and understanding of these individual differences.

INTRODUCTION

In many mathematics courses around the world Computer Algebra Systems (CAS) now take their place among the smorgasbord of tools available for doing, teaching and learning mathematics. CAS is arguably the most complex tool that any students are expected to use at school or elsewhere, and so it is important to know whether mastery is realistically within the capabilities of students and teachers. The work reported in this paper is also motivated by the need, when CAS is used in teaching, to monitor students' progress and plan teaching across a continuum of knowledge and skills from machine utility, through technical facility, to mathematical facts and concepts. In particular, this paper focuses on the technical difficulty that is experienced when student, machine and mathematics connect.

The data was collected throughout 2002 from a class participating in the trial (CAS-CAT project: website http://www.edfac.unimelb.edu.au/DSME/CAS-CAT) of a new tertiary-preparation mathematics subject where CAS calculators were available at all times, including for examinations. The students had used CAS throughout 2001, and were preparing in 2002 for their final year examinations. The experienced and highly motivated teacher was as new to CAS use as the students. She taught relevant CAS features, actions and strategies by demonstrating through a view screen. Students were also encouraged to suggest efficient syntax or command sequences. This paper presents the results and views of students in their second year of using CAS.

The paper shows that, regardless of mathematical ability and with good teaching, effective CAS use is within the capability of students who are willing to overcome the initial hurdles. The findings that most students developed good technical facility also emphasise the individual nature of students' response to CAS and the need for monitoring, to inform teaching, if students are to develop automated technical skills with CAS.

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LEARNING TO MAKE EFFECTIVE USE OF CAS

In mathematics courses for which the use of CAS has been accepted, students need to learn to operate the technology effectively and to integrate it with their repertoire of techniques for doing and learning mathematics. That this is not a simple process has been acknowledged. For example, Guin and Trouche (1999) outline a complex process which they call 'instrumental genesis. They claim this process is required to turn the CAS machine into a 'mathematical instrument' that a student can use skillfully. Lagrange (1999) points out that learning to use the technology of a CAS presents new, additional challenges for students. In his study, many students who felt that they were competent CAS users actually had difficulty using basic home screen commands. Drijvers (2000) also describes key obstacles, including technical difficulties that impede students' use of CAS. Guin and Trouche (1999) comment that the syntactic requirements of CAS can be demanding and have to be memorised. Technical difficulties and the distraction of correcting syntax errors should not complicate students' focus on conceptual learning. Students' use of the technical facilities of CAS needs to become automated, especially when CAS is used for learning mathematics. It is important that these difficulties are recognised and addressed by teaching.

Students' effective use of CAS will not be determined only by these cognitive issues. Affective factors (Pierce and Stacey, in press) will determine the purposes for which students use CAS (e.g. strictly functional use to get answers, or including pedagogical use to explore) as well as the effort they make to overcome the many initial obstacles. Affective factors may determine the effort which students put into the process of learning to use CAS correctly, efficiently, even automatically and, in particular, the degree to which their own mathematical habits and learning strategies are changed as a result of the new possibilities afforded by the availability of CAS.

To expedite the process of instrumental genesis external guidance is necessary. Trouche (2003) argues that this necessity is rarely taken into account. He explains the need for carefully planned teaching episodes that consider a number of dimensions in what he terms 'orchestrated instrumentations'. Designing suitable learning experiences requires an understanding of the mathematics, the CAS, the students and impediments to their positive interaction. The reactions and interactions at the interface between students, mathematics and CAS add a new dimension to mathematics classrooms, which are studied in this paper.

NUMERIC DATA COLLECTION AND RESULTS

The data reported in this paper was collected from 30 students during their second year of working with CAS. A survey instrument was administered on four occasions: February (the beginning of the school year), May, August and October (shortly before final examinations); a 'Basic CAS Skills' test was conducted in October; and the experienced classroom teacher was interviewed. The 'Use of CAS Questionnaire' consisted of the same 36 Likert scale items each time plus up to 6 open questions.

Students were asked to reflect on their experience during the previous week. The open questions always offered the students the opportunity to note any technical difficulties they were experiencing and anything else they would like us (or future students) to know about their experience of doing and learning mathematics with CAS available. The closed items were separated into two sections. The first 16, included below as Table 1, identified CAS features or actions along with 5 response options scaled from 'Very Hard' (scored as 1) to 'Very Easy' (scored as 5) plus the possibility 'Not Used' (omitted from averages).

CAS feature or action	
1. Using CAS symbols like / for division and ^ for 'raise to power'	9. Changing between symbols and tables
2. Bracketing expressions as required	10. Viewing the important parts of my graphs
3. Remembering what names like TRNS mean.	11. Setting table start and increments
4. Using CAS syntax eg. Substitute(*3x*,*x*=4)	12. Finding all of my answers on the screen
5. Working with letters other than *x*	13. Scrolling through my working steps
6. Defining or using functions	14. Obtaining exact, approximate, surd or decimal solutions
7. Finding my way through the menus.	15. Copying graphs from the screen
8. Changing between symbols and graphs	16. Interpreting CAS symbols as ordinary maths

Table 1: Survey items relating to the technical aspect of CAS use

Boxplots of the average class results for the technical issues section of the survey data are included below as Figure 1. These graphs indicate that, by the end of their second year of working with CAS, most students were confident and half of the students felt that using CAS was either easy or very easy. However, no item was indicated as Very Easy or Easy by every student. Students were most confident about item 1, but some other items had up to 13 percent of students who found them to be Hard or Very Hard. Students indicated least confidence in items 2, 6, 8, 10 and 16. For example, almost half the students indicated that they found item 8 (*Changing between symbols and graphs)* to be Hard or OK (score 2 or 3). The data shows some improvement on average over the year, but new challenges arose at every stage.

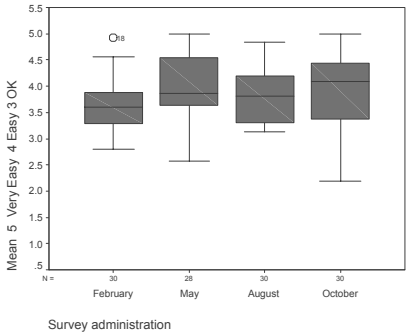

Figure 1: Students' average rating of their technical facility throughout the year.

The ‘Basic CAS Skills’ test in October required students to use their CAS to perform 16 mathematical tasks such as entering expressions with algebraic fractions and parameters, evaluating integrals and finding the solutions of simultaneous equations. These items covered the essential tasks that students needed for their imminent, high-stakes, final examinations. Students recorded the number of attempts required before achieving ‘no syntax errors and a correct result’. This task was undertaken as a formative revision ‘test’ in class with immediate feed back from the teacher via the view screen. We recorded the number of tasks which students completed using correct syntax on their first attempt. These results, illustrated in Figure 2, show that most students had few problems with CAS syntax on familiar tasks, with all students performing the majority tasks correctly at first attempt. It is not surprising that most students made one or two slips when lengthy or composite syntax was required.

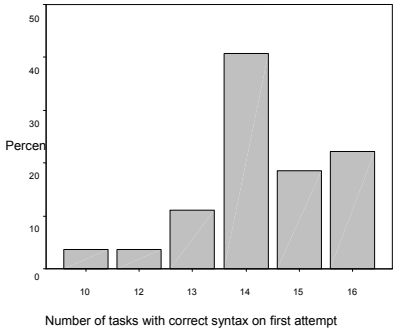

Figure 2: Number of tasks correct on first attempt (max 16).

SIX STUDENTS’ EXPERIENCES

The issue of difficulties with the technical aspect of effective use of CAS is not simple. A broad grouping on students as being of ‘high’, ‘middle’ or ‘low’ year 12 mathematics achievement was based on the judgement of the class teacher. In this study the teacher, who had at least 10 years experience of teaching the equivalent course without CAS available, was well qualified to make this assessment. Figure 3 below illustrates details of 6 students (pseudonyms used) chosen because they provide clear evidence that level of technical facility is *not* only explained by mathematical ability. Technical facility in Figure 3 is reported in three loose groupings, related to inputting information, using commands and interpreting results.

The second set of closed items on the ‘Use of CAS Questionnaire’ included 20 statements describing personal approaches to using CAS. This included statements such as ‘I use CAS to try out ideas’, ‘I like using CAS’ and ‘I can do harder maths with CAS’. Students were asked to indicate, on a 3-point scale, how often each description applied to them in the previous week. This information was used to

classify students' attitude towards CAS and how they used it. Using CAS in a limited manner only, to do some questions faster, or when the teacher directed, but not to initiate exploration was classified as 'low usage'. High use included exploring ideas.

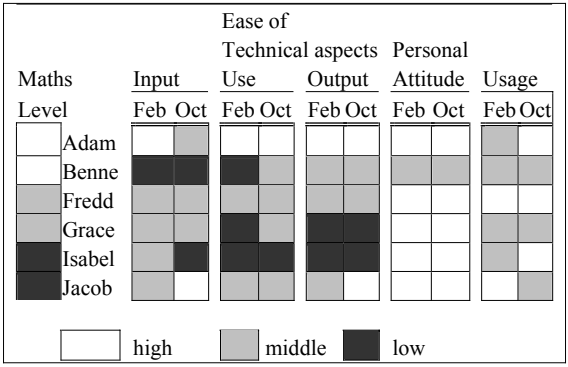

Figure 3: Mathematical, Technical and Personal aspect levels for six students

Adam, a very high achieving mathematics student, was also strong in each aspect of Effective use of CAS. The class teacher described his use of CAS as elegant. He began the year with confidence, writing in February: "Easy, feel confident, interested. It's [CAS] a very good machine, making maths a lot quicker and easier to understand." Following an important test in May, he indicated that he had had no difficulties but rather found CAS to be "lots of help" throughout the task. At the end of the year his advice to future students was: "Don't be confused by the syntax. Try to remember what each function does and use it to your advantage".

In contrast Benne, who was also a very strong mathematics student, did not like using CAS and, despite the evidence of the Basic CAS Skills test, when he completed each task correctly at his first attempt, never said that he found CAS easy. In February he indicated that CAS did not make any area of mathematics easier nor did it help him in his understanding. He preferred to use pen and paper. By August he indicated that he did make some use of CAS: "I make sure that I know how to do the problem by hand before using CAS and use CAS to check the answer." The limited value that he placed on CAS is clear. He did not use CAS to explore maths or try out ideas: " Not enough time to…actually can't be bothered." Benne's mildly sarcastic comment to future students indicates a reluctant acceptance of the technology. "It's good. Make sure you can do stuff by hand as well but know how to use all features of CAS- know how to turn it on/ take off the cover."

Fredd, a middle ability mathematics student, felt strongly positive towards about the role of CAS in learning and made strategic use of its facilities. This was despite continued technical difficulties. In February he wrote: "I like CAS because it is easier." During the May test he experienced difficulty "when putting functions into the graph menu, [format] often has to be varied." In August, he also commented that

he had difficulty “ensuring that brackets [parentheses] are right in long equations in CAS menu.” This did not discourage Fredd. He wrote: “I use CAS for ‘hard’ questions to often check answers with it as a safe guard. It is a waste of time for easier questions. … We use CAS to explore broader ideas, not just the single or specific questions.” In October his advice to future students emphasised the tension between needing mathematical knowledge to use CAS effectively and the role of CAS in expanding mathematical understanding. Fredd’s technical difficulties don’t seem to be important to him. “Go for it, if you are not strong in by-hand skills. Good if you understand what you use it [CAS] for. If not it isn't useful for you. It can dramatically expand your understanding. If you are not strong in maths the CAS program is not as beneficial.”

Grace, another middle band mathematics student, experienced technical difficulty but retained a positive attitude. In February she wrote: “I have found that for some topics CAS has been really helpful.” In August she still found “using brackets in appropriate places frustrating” but “I will try to do the question both by hand and then on CAS and this helps me to understand the answer I got.” Grace’s advice to future students indicates an awareness of her technical difficulties along with thoughtful reflection on the role of CAS. “Always plan what you are going to do first in your head and not just key in anything - understand what you are doing. Keep practising your by-hand skills - this is one disadvantage of the CAS because I find factorising etc. difficult now because I have forgotten how to do a lot of simple maths. Persist with the CAS, it becomes easier with time.”

Isabel, a low ability mathematics student, also experienced considerable difficulty managing the technical aspects of CAS. Against these odds she remained positive about both mathematics and the use of CAS. With a positive start to the year, in February, Isabel wrote that she used CAS “for lots of things” and that “differentiating is made easier with CAS”. In the important May test Isabel had trouble with syntax errors: “It is annoying how when you get errors and then it doesn't actually tell you where you have gone wrong” and setting an appropriate graph window: “It is annoying if you can't see the graph and can't find it then you have to search for ages.” She also found CAS unhelpful when she needed to “use lots of letters.” In August, Isabel still found working with “letters other than x a nuisance” but, despite such fundamental technical difficulties, she still valued to facility of CAS to explore mathematics. “I find it easier just to put an eq[uation] into the CAS and then play around till I find an answer that looks right. Most times I get it. Also long questions and diff[erentiation]/ antidiff[erentiation] are so good on CAS.” She wrote “The CAS is good [be]cause it makes maths easy.” Isabel’s positive advice to future students acknowledges that using CAS is not trivial :” DO IT! It's really good - but it can be confusing and kind of hard as well. Find the shortcuts on the different menus of the CAS before you do anything else - they are the best things. Do by hand as well.”

Jacob was a low ability mathematics student who, according to the class teacher, would not have tackled the equivalent mathematics subject without CAS. He felt

empowered by CAS. In February, his cautiously optimistic comment reflects the technical difficulties he has experienced: "Often frustrated, occasionally it's a good thing." After the important May test he wrote that he had found CAS helpful for most tasks. He continued, "Without CAS, I would be lost; it helps me understand maths in general. With CAS, I can do maths confidently!" His reflection in August outlines the degree to which he has come to value CAS. "I've never been good in the maths department, especially the 'hard' questions. Using CAS enables me to understand maths and have a 'crack' at questions I would usually leave!!! Without CAS I struggle … so I use it for everything. It's incredibly helpful and a great learning tool. I can do harder maths with CAS. I can now do maths with confidence and an understanding of what I'm trying to do." Jacob's advice to future students was: "Learn to do problems both ways, by hand and by CAS; that way you can check your answers and fully understand what's going on."

DISCUSSION

These students serve to remind us of the individual variation in students' mastery of this technology. Adam's combination of positives, for every aspect of his CAS use, certainly allowed him to employ it to the advantage of both his doing and his learning of mathematics. Benne, who preferred to work by hand, did not automate the technical aspect sufficiently for it to become easy and free him to focus on the mathematic, as he did without CAS. Fredd did not allow his technical difficulties to affect his attitude or constrain the scope of his CAS use. Grace, who was cautiously positive about the role of CAS, saw technical mastery as requiring persistent effort. She was concerned about the effect of reliance on CAS and therefore perhaps did not 'practise' CAS skills on 'simple' mathematics. Isabel, who had only low mathematics ability, continued to experience fundamental technical difficulties but, despite this, felt empowered by CAS in both doing and learning mathematics. It seems that Jacob, although a low ability mathematics student, succeeded in overcoming his early technical difficulties. From his comments, it seems likely that he made a great deal of use of CAS and that, because he saw it as an essential partner in his mathematics progress, he was prepared to make the effort to become a competent user.

The nature of the common difficulties reported, in particular the use of brackets, are similar to those described by Drijvers (2001) and Lagrange (1999). Students, in their initial use of CAS, often report frustration at both the need to correctly use brackets and to efficiently move between representations. Within a generally positive climate, it was interesting to see that, for some students, fundamental difficulties persist well into their second year of CAS use, even for skills that are likely to improve with practice. The resistance of some able students to mastering these actions is especially interesting, in this context, because this well motivated class, preparing for high stakes examinations, was taught by a teacher who was excited by the opportunities afforded by CAS. Kendal and Stacey (2001) report the impact of teachers' underlying beliefs and values with respect to mathematics on what they privilege in their teaching of mathematics in a CAS active environment. In this study we see the

influence of students' attitudes, based on their beliefs about how they learn mathematics, on their adoption of CAS as an instrument for doing or learning mathematics. These results are consistent with those of the authors' previous study undertaken with first year undergraduate students (Pierce and Stacey, in press), where we also observed able students who used CAS effectively, able students who did not want to use CAS, along with previously weak mathematics students who became able exploiters of the facility of CAS. In this study, the Effective Use of CAS framework (Pierce and Stacey 2002) proved sufficient for highlighting the technical and personal aspects of students' thinking that impacted on each student's use of CAS.

CONCLUSION

The evidence presented in this paper emphasises the importance of giving due consideration to the extra layer of complexity which developing effective use of CAS can add to the mathematics classroom. Both the teachers' and the students' attitudes have an effect. Although obstacles were encountered, students generally were well able to master the required technical aspects of CAS whilst they learned a demanding mathematics subject. The level of technical difficulty that students may have in using CAS and the degree to which this may present an obstacle to their mathematical learning is not predictable on the basis of their 'conventional' mathematical ability. To plan appropriate teaching for the effective use of CAS it is therefore important to undertake some monitoring of this dimension of students' progress.

References

Drijvers, P. (2000). Students encountering obstacles using CAS. *International Journal of Computers for Mathematical Learning, 5,* 189-209

Guin D. & Trouche, L. (1999). The complex process of converting tools into mathematical instruments: The case of calculators. *International Journal of Computers for Mathematical Learning, 3*, 195-227.

Kendal, M., & Stacey, K. (2001). The impact of teacher privileging on learning differentiation with technology. *International Journal of Computers for Mathematical Learning. 6*, 143-165.

Lagrange, J.-B. (1999b). Complex calculators in the classroom: theoretical and practical reflections on teaching pre-calculus. *International Journal of Computers for Mathematical Learning, 4*, 51-81.

Pierce R. & Stacey, K. (2002). Monitoring Effective Use of Computer Algebra Systems in B. Barton, K. Irwin, M. Pfannkuch & M. Thomas (Eds.) *Mathematics Education in the South Pacific* (Proceedings of the 25th Annual conference of the Mathematics Education Research Groups of Australasia, Auckland, Vol 2 pp 575 -582.) (Refereed paper)

Pierce, R. & Stacey. K. (in press). A framework for monitoring progress and planning teaching towards effective use of computer algebra systems. *International Journal of Computers for Mathematical Learning.*

Trouche, L. (2003). Managing the complexity of human/machine interaction in a computer based learning environment(CBLE): Guiding student's process command through instrumental orchestrations. Paper presented at 2003 Symposium on Computer Algebra in Mathematics Education, France. Retrieved January 11, 2004, from: http://ltsn.mathstore.ac.uk/came.