THE EFFECT OF STUDENTS' ROLES ON THE ESTABLISHMENT OF SHARED MEANINGS DURING PROBLEM SOLVING

Konstantinos Tatsis   **Eugenia Koleza**

Department of Primary Education   Department of Primary Education

University of Ioannina, Greece   University of Ioannina, Greece

The analysis of interaction among students is becoming very important in mathematics education, especially among the scholars who view knowledge as socially constructed and negotiated. One of the features of human interaction is the roles that people perform while interacting. Using a symbolic interactionist perspective, we analyzed the evolution of students' roles in time and their effect on the establishment of shared meanings.

INTRODUCTION

It seems to be a common faith between researchers in mathematics education that interactive mathematical activities provide students with learning possibilities that extend "beyond the realms of memorized procedures" (Wood 1994, p. 149). In other words, a real acquisition of knowledge occurs while the individual engages in a certain discourse[1]. A considerable amount of recent research in mathematics education is dedicated to the analysis of the main factors that comprise discourse: the participants engaged, the language used and the rules that control the interactions[2]. Although there seem to exist many analyses concerning language (Pirie and Schwarzenberger, 1988, Dekker and Elshout-Mohr, 1998, Forrester and Pike, 1998, Stacey and Gooding, 1998, Dreyfus, Hershkowitz and Schwarz, 2001), how it affects the process of interaction (Gómez and Rico, 1995, Ward and Jacobs, 2000) and what rules regulate the interaction (Yackel and Cobb, 1996, Sfard, 2000, Yackel, Rasmussen and King, 2000, Yackel, 2001), little work is done on the acts of the participants themselves, especially on how these acts reflect their wider context or how they affect the interaction or the acquisition of knowledge itself (César, 1998, Carvalho and César, 2001, Rowland, 2002). Our research, having as its

---

[1] "… the word *discourse* has a very broad meaning and refers to the totality of communicative activities, as practiced by a given community" (Sfard, 2000, p. 160).

[2] A recent case study we have conducted (Tatsis and Koleza, 2002) focused on all these factors: we investigated the type of language used, the norms that controlled the interactions and the roles that were adopted by university students while they worked collaboratively.

---

main focus the participants' acts, their distribution across the interactions and their effect on the creation of shared meanings intended to answer the following questions:

a) What are the actual roles[3] that students perform while collaborating to solve a mathematical problem?

b) What is the evolution of these roles in time?

c) What is the effect of that role playing in the establishment of shared meanings by the students?

THEORETICAL FRAMEWORK

Symbolic interactionism is a sociological theory introduced by Mead (1934) and Blumer (1969) and elaborated by Goffman (1961, 1971, 1972) among others. Like its name suggests, this theory considers vital the role of symbols for the process of interactions; it is through symbols that people establish shared meanings and define the situation they are involved. Language is the most important symbol, although other non-verbal symbols can sometimes be the object of investigation. The individual is not treated as a passive receiver of society's influences, but as an active participant who takes part in the formulation and negotiation of knowledge during the process of symbolic interaction. This process involves several inter-connected features; a basic one, that may be said to include all the rest, is the individual's *behavior* or *performance*, which is defined as "all the activity of a given participant on a given occasion which serves to influence in any way any of the other participants." (Goffman, 1971, p. 26)

The analysis of performance can be done by the use of role theory, which uses two basic models[4] for its analyses: the dramaturgical model treats the individual as an actor, who presents himself to others and tries to guide and control their impression of him; the game model considers human interactions as a sort of a game which places constraints and rules for participants' behavior, and gives them the chance to employ various strategies in order to achieve their goals. We treat these two models as complementary rather than contradictory, because they both share one of the basic assumptions of symbolic interactionism: people's acts are the product of interpretation of other's acts.

One of the concepts that proved very helpful on our attempt to explain particular characteristics of performances is *face*, which is "the positive social value a person effectively claims for himself by the line others assume he has taken during a particular contact." (Goffman, 1972, p. 5). During each

---

[3] Role is defined as "A behavioral repertoire characteristic of a person or a position" (Thomas and Biddle, 1966, p. 11)

[4] Both models owe their development to Goffman's (1961, 1971, 1972) work.

---

interaction some *face work* takes place: as soon as the person realizes all the social norms concerning his position, and the possible interpretations of others upon his acts, he employs his face saving strategies.

METHODOLOGY

Subjects of our research were 40 undergraduate students of the Department of Primary Education of the University of Ioannina in Greece. The students were asked to choose a partner, so 20 pairs were formed. The only instructions given to the students were that they should verbalize every thought they make and that they should try to cooperate to solve the problems posed. All sessions were tape-recorded by the observer, whose interventions were the fewest possible.

Once we got at hand the transcripts, we were engaged in the process of coding the data[5]: firstly, we labeled all the verbal acts of each subject, according to the following labels[6]: a) shows certainty, b) shows uncertainty, c) shows agreement, d) shows disagreement, e) makes suggestion, f) asks for suggestion, g) gives opinion, h) asks for opinion, i) gives information, j) asks for information. We also monitored each student's acts, his partner's *responses* to these acts and the *effect* of these acts in the process. Then we examined the degree of conformity to social and sociomathematical norms that these acts exhibited. All these elements assisted us in the categorization of the mentioned labels. The following categories emerged: a) collaborator (i.e. a person who always asked for her partner's opinion before proceeding) b) contributor (i.e. a person that made many suggestions) c) elaborator (i.e. a person who gave information concerning her suggestions whenever it was possible), d) conciliator (i.e. a person who rarely insisted on a suggestion once it was withdrawn). These categories were dimensionalized in the sense that a person's acts could range from collaborative to non-collaborative, or a person could be indifferent, and so on. The combination of the above categories provided us with enough information to describe each student's role. Finally, in order to study the effect of that role playing on the establishment of shared meanings, we observed the processes by which new meanings were introduced, elaborated and accepted (or abandoned) and then studied how these processes were connected with particular combinations of roles. A brief illustration of our analytic process shall be provided in the sample analysis that follows.

SAMPLE ANALYSIS

The following excerpt is taken from a "girl-girl" pair's first session, once they were assigned the T-shirt Problem (see Appendix). Next to each student's turn,

---

[5] We mainly followed Strauss and Corbin's (1990) methodology for coding our data.

[6] The labels were adapted from Bales (1966).

lies the label of the act and next to it the degree of this act's conformity to a social or sociomathematical norm.

Verbal acts	Labels	Conformity
1. A. Do you understand what we have to do?	Asks for opinion.	Social norm[7] (high)
2. B. Yes.	Shows agreement.	
3. A. Count the squares and...	Makes suggestion.	
4. B. Number them?	Asks for information.	
5. A. Shall we count them? One side is one two three four and here is one two three four. Let's check if… I believe that these two triangles are equal, because… Because, these two angles, hold on…	a. Asks for opinion. b. Gives information. c., d., e. Makes suggestion and shows uncertainty.	a. Social norm (high) c. Social norm (medium) d. Social norm (low)
6. B. Are these ones angles?	Asks for information.	
7. A. Yes. Vertical angles. So, this side is the same with that, one common side and one angle… What are you thinking? So, this is the circle's center…	a., b., c. Gives information. d. Asks for opinion. e. Makes suggestion.	d. Social norm (medium)
8. B. Hm.	Shows uncertainty.	
(Middle pause)		
9. B. And what are we going to say to him about the drawing of the circle?	Asks for suggestion.	Social norm (medium)

---

[7] The only social norm found in the excerpt is the one that refers to the respect that one has to show for her partner's opinion. High level of conformity was expressed by questions concerning the partner's understanding and/or asking for her opinion. Low level of conformity was expressed by suggestions made in the affirmative and in first person singular. Medium level of conformity was expressed by all verbal acts that contained an element of collaboration (usually the pronoun "we"), but could not be classified at the high level for various reasons.

10. A. Hm. Shall we say that first?	a. Shows uncertainty. b. Makes suggestion and asks for opinion.	Social norm (high)
11. B. Yes.	Shows agreement.	
12. A. OK, it's a square with ten small squares in each side. How are we going to write the instructions? The design consists of a circle…	a. Makes suggestion and shows certainty. b. Asks for suggestion. c. Makes suggestion.	b. Social norm (medium)
13. B. And two triangles.	Makes suggestion.	
14. A. So, we shall begin like this…	Gives opinion.	Social norm (medium)
15. B. Hm.	Shows uncertainty.	

A first observation one can make is that student A made most suggestions, sometimes asked for her partner's opinion and always gave sufficient information. She also seemed to show a high level of conformity to the social norm of collaboration[8]; a closer look though, revealed that student A's adherence to that norm was sometimes superficial: in 5, 7 and 12 immediately after posing a question, she proceeded without waiting for her partner to reply. One might say that from the one hand her aim was to maintain her face as a collaborative partner, while on the other hand she wanted to avoid a possible threat to her competitive face[9] by her partner, by eagerly uttering her own suggestion. Student B made almost no suggestions (13 contains a suggestion already made by her partner) and was collaborative in the sense that she listened to her partner's suggestions, expressed her agreement or uncertainty and asked for information. Thus, one cannot conclude that she played a passive role in the episode: her hesitant replies seemed to lead her partner to clarify or elaborate her suggestions.

The first meaning introduced in the excerpt was "numbering the squares" (by student A) which became immediately a shared meaning, since student B accepted it in 4 without showing uncertainty or asking for information. The next meaning, introduced by student A again, was "equal triangles" (5); this time her partner seemed reluctant to accept it: firstly she asked for information (6), then she showed uncertainty (8) and although in 13 she uttered "two triangles" she

---

[8] With the exception of 5 d. student A always used the pronoun "we" in her expressions.

[9] A threat to her competitive face might be consisted of a more effective suggestion, or of a request for information that she could not handle.

did not use the adjective "equal". This is an example of a meaning that did not become a shared one.

CONCLUSIONS

The cross-examination of the protocols led us to four basic role categories: "the collaborative initiator", "the dominant initiator", "the collaborative evaluator" and "the insecure conciliator". The collaborative initiator demonstrated the highest level of conformity to social and sociomathematical norms; she made many suggestions, gave information whenever necessary and was ready to withdraw a suggestion in order to maintain the collaboration. The dominant initiator made many suggestions, but rarely asked for her partner's opinion; she elaborated her proposals, but was reluctant to withdraw a suggestion. The collaborative evaluator made relatively few suggestions, either because she felt uncertain of her skills or because of her partner's behavior. The insecure conciliator made almost no suggestions; she expressed the lowest level of conformity to most norms, as her remarks usually showed agreement to her partner's acts, without any sort of evaluation.

The above roles changed very slightly in the course of the three meetings. It seems that once the roles were established, both students tried to maintain them; this may be partially attributed to a face maintaining strategy. Eventually, some roles changed during the three meetings: some insecure conciliators switched to collaborative evaluators; this may be attributed to their partners' behavior, since all were collaborative initiators. The establishment of shared meanings was also affected by students' roles; the pairs that consisted of two collaborative initiators produced the most shared meanings; a collaborative initiator with a collaborative evaluator also produced many shared meanings, but not as many as the previous combination. The less shared meanings were established in the pairs consisting two dominant initiators and in all pairs consisting an insecure conciliator.

References

Bales, R. F. (1966). Task Roles and Social Roles in Problem-Solving Groups. In B. J. Biddle, E. J. Thomas (Eds.), *Role Theory: Concepts and Research*. New York: John Wiley & Sons, Inc., 254-262.

Blumer, H. (1969). *Symbolic Interactionism: Perspective and Method,* Englewood Cliffs, NJ: Prentice-Hall.

Carvalho, C., César, M. (2001). Peer Interactions and Statistics Learning. In M. van den Heuvel-Panhuizen (Ed.), *PME 25 Proceedings*. The Netherlands: Freudenthal Institute, Faculty of Mathematics and Computer Science, Utrecht University, vol. 2, 217-224.

César, M. (1998). Social Interactions and Mathematics Learning. *MEAS 1 Proceedings*. Nottingham: Nottingham University, 110-119.

Dekker, R., Elshout-Mohr, M. (1998). A Process Model for Interaction and Mathematical Level raising. *Educational Studies in Mathematics, 36(3),* 303-314.

Dreyfus, T., Hershkowitz, R., Schwarz, B. (2001). The construction of abstract knowledge in interaction. In M. van den Heuvel-Panhuizen (Ed.), *PME 25 Proceedings.* The Netherlands: Freudenthal Institute, Faculty of Mathematics and Computer Science, Utrecht University, vol. 2, 377-384

Forester, M. A., Pike, C. D. (1998). Learning to Estimate in the Mathematics Classroom: A Conversation-Analytic Approach. *Journal for Research in Mathematics Education, 29 (3),* 334-356.

Goffman, E. (1961). *Encounters: Two Studies in the Sociology of Interaction.* Indianapolis: Bobbs-Merrill.

Goffman, E. (1971). *The Presentation of Self in Everyday Life.* Harmondsworth, Middlesex: Penguin University Books.

Goffman, E. (1972). *Interaction Ritual: Essays on Face-To-Face Behaviour.* Harmondsworth, Middlesex: Penguin University Books.

Gómez, P., Rico, L. (1995). Social interaction and mathematical discourse in the classroom. In L. Meira, D. Carraher (Eds.), *Proceedings of the 19th PME Conference.* Recife: Universidade Federal de Pernambuco.

Mead, G. H. (1934). *Mind, Self and Society.* Chicago: University of Chicago Press.

Pirie, S. E. B., Scharzenberger, R. L. E. (1988). Mathematical discussion and mathematical understanding. *Educational Studies in Mathematics, 19(4),* 459-470.

Rowland, T. (2002). Pragmatic Perspectives on Mathematics Discourse. In J. Novotná (Ed.), *CERME2: European Research in Mathematics Education II.* Prague: Charles University, 408-419.

Schiffrin, D. (1994). *Approaches to Discourse.* Oxford: Blackwell.

Sfard, A. (2000). On reform movement and the limits of mathematical discourse. *Mathematical Thinking and Learning, 2(3),* 157-189.

Stacey K., Gooding, A. (1998). Communication and Learning in Small-Group Discussions. In H. Steinbring, M. G. Bartolini Bussi, A. Sierpinska (Eds.), *Language and Communication in the Mathematics Classroom*. Reston, VA: National Council of Teachers of Mathematics, 191-205.

Strauss, A., Corbin, J. (1990). *Basics of Qualitative Research. Grounded Theory Procedures and Techniques.* Newbury Park: Sage Publications.

Tatsis, K., Koleza, E. (2002). Language as a Communicative and Interpretive Tool in Mathematical Problem Solving. *2nd International Conference on the Teaching of Mathematics Proceedings,* http://www.math.uoc.gr/~ictm2/Proceedings/pap262.pdf

Thomas, E. J., Biddle, B. J. (1966). The Nature and History of Role Theory. In B. J. Biddle, E. J. Thomas (Eds.), *Role Theory: Concepts and Research.* New York: John Wiley & Sons, Inc., 3-19.

Ward, C. S., Jacobs, G. M, (2000). Analyzing Student-Student Interaction from Cooperative Learning and Systemic Functional Perspectives. *Electronic Journal of Science Education, 4 (4),* http://unr.edu/homepage/crowther/ejse/jacobsetal.html

Wood, T. (1994). Patterns of Interaction and the Culture of Mathematics Classrooms. In S. Lerman (Ed.) *Cultural Perspectives on the Mathematics Classroom.* Dordrecht: Kluwer Academic Publishers, 149-168.

Yackel, E., Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education, 27(4),* 390-408.

Yackel, E., Rasmussen, C., King, K. (2000). Social and sociomathematical norms in an advanced undergraduate mathematics course. *Journal of Mathematical Behavior, 19 (3),* 275-287.

Yackel, E. (2001). Explanation, justification and argumentation in mathematics classrooms. In M. van den Heuvel-Panhuizen (Ed.), *PME 25 Proceedings.* The Netherlands: Freudenthal Institute, Faculty of Mathematics and Computer Science, Utrecht University, vol. 1, 1-9.

Appendix

T-shirt Problem

The design below is going to be used on a T-shirt. You accidentally took the original design home, and your friend, Chris, needs it tonight. Chris has no fax machine, but has a 10 by 10 grid just like yours. You must call Chris on the telephone and tell him precisely how to draw the design on his grid. Prepare for the phone call by writing out your directions clearly, ready to read over the telephone.