

THE GENESIS OF SIGNS BY GESTURES. THE CASE OF GUSTAVO

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This paper highlights the role of gestures in communicating and particularly in thinking in mathematics. The research interest is on the relation between the use of gestures and the birth of new perceivable signs. This link is shown through the description of a concrete example, referring to a discussion among 8th grade students around a geometrical problem in 3D, to be solved without the use of devices and paper and pencil. It is interesting to observe the progression in the construction of the solution, obtained with the introduction of new signs from gestures, and at the end even of a common tool used by children, the plasticine.

INTRODUCTION

In the last years psychologists have shown a deep interest on the analysis of gestures and their role in the construction of meanings. More recently, gestures became also relevant in the field of mathematics education, in order to show a strong relation not only with speech, but with the entire environment where the genesis of mathematical meanings takes place: context, artefacts, social interaction, discussion, and so on. Since math is an abstract matter, it often requires signs to be made somehow perceivable by students, the abstract becoming more and more concrete to them. Many times students need to see, to touch, and to manipulate, and as a consequence, the environment plays a crucial role in learning math.

This paper shows the link between the use of gestures and the birth of new perceivable signs through a concrete example. To reach this aim, we use a theoretical framework made of different components, coming from mathematics education, psychology, neuroscience and semiotics, and presented at the Research Forum on gestures at the current PME (Arzarello *et al.*, 2005). Three parts constitute the paper. We sketch some ideas of the theoretical framework in a first paragraph. The second paragraph presents an example of a protocol analysed through our theoretical tools. In the last part, conclusions and further problems are introduced.

THE THEORETICAL FRAMEWORK

Some researchers from psychology and mathematics education claim that gestures play an active role in thinking, intending communicating and thinking not as mutually exclusive functions (McNeill, 1992; Goldin-Meadow, 2003). So gesturing is useful to listeners to communicate, and to speakers to think. To exemplify, just think of people speaking on the phone: they are completely conscious of the fact that their

interlocutor is not in front of them, but all the same they gesture, as well as the listener would be there. Even a subject alone, doing an activity such as studying, could gesture to give expression to her thoughts. Briefly speaking, we can gesture for ourselves or for others, and gesture is both a mean of communication and of thinking. Gesture in fact can contribute to create ideas:

According to McNeill, thought begins as an image that is idiosyncratic. When we speak, this image is transformed into a linguistic and gestural form. ... The speaker realizes his or her meaning only at the final moment of synthesis, when the linear-segmented and analyzed representations characteristic of speech are joined with the global-synthetic and holistic representations characteristic of gesture. The synthesis does not exist as a single mental representation for the speaker until the two types of representations are joined. The communicative act is consequently itself an act of thought... It is in this sense that gesture shapes thought. (Goldin-Meadow, 2003; p. 178)

Within the perspective of psychology, we refer to the so-called Information Packaging Hypothesis (IPH): Alibali, Kita & Young (2000) describe it as the way gesture is involved in the conceptual planning of the messages. IPH concerns the so called representational gestures (Kita 2000), namely the iconic and the abstract deictic gestures: an iconic gesture represents an entity or a phenomenon, a movement, a shape, and so on; a deictic gesture points to an object (Mc Neill, 1992). This object can be a physical thing or an abstract entity, so deictic gestures can be divided in concrete or abstract (Kita, 2000). According to the IPH, the production of representational gestures seems particularly important, since it helps speakers organise spatio-motoric information into packages suitable for speaking. In such a sense gesture explores alternative ways of encoding and organising spatial and perceptual information. Spatio-motoric thinking (constitutive of representational gestures) provides an alternative informational organisation that is not readily accessible to analytic thinking (constitutive of speaking organisation). Analytic thinking is normally employed when people have to organise information for speech production. On the other side, spatio-motoric thinking is normally employed when people interact with the physical environment, using the body (interactions with an object, locomotion, imitating somebody else's action, etc.). This kind of thinking can be applied even to the virtual environment that is created as imagery.

Gesture can be discussed also from a neuroscientific point of view. In fact some studies from neuroscience argue that there is no inherent distinction between thought and movement at the level of the brain; both can be controlled by identical neural systems (Ito, 1993). Therefore, concepts and ideas can be manipulated just as they were body parts in motion. The 'motor system' is thus a complex computational network that controls and directs the brain's circuitry or internal symbols: counting, timing, sequencing, predicting, planning, correcting, attending, patterning, learning and adapting (Leiner *et al.*, 1993). Indeed, gestures that accompany language may facilitate thought itself. With the embodied mind Seitz (2000) introduces a fresh paradigm for thinking about the relation of movements to thoughts, in which the

boundaries between perception, action, and cognition are porous. Thought, action, and perception are indissolubly tied. Thinking of embodied activities, although humans may be best characterised as symbol-using organisms, symbol use is structured by action and perceptual systems that occur in both natural environments and artifactual contexts: “[...] body structures thought as much as cognition shapes bodily experiences.” (Seitz, 2000).

From another viewpoint, gestures can be seen as signs, as pointed out by Vygotsky (1997: 133):

A gesture is specifically the initial visual sign in which the future writing of the child is contained as the future oak is contained in the seed. The gesture is a writing in the air and the written sign is very frequently simply a fixed gesture.

As a consequence, semiotics is a useful tool to analyse gestures, but within a wider frame, which involves also their cultural and embodied aspects. Such a wider analysis has been developed in mathematics education with the introduction of the notion of *semiotic means of objectification* (Radford, 2003) that produce the so-called *contextual generalisation*: a generalisation referring still heavily to the subject’s actions in time and space, within a precise context, even if he/she is using signs who could have a generalising meaning. In contextual generalisation, signs have a twofold semiotic nature: they are becoming *symbols* but are still *indexes*. We use these terms in the sense of Pierce (1955): an index gives an indication or a hint on the object, like an image of the Golden Gate makes you think of the town of S. Francisco. A symbol is a sign that contains a rule in an abstract way (e.g. an algebraic formula).

In light of the results stated above, our research hypothesis is that meanings construction is supported by a dynamic evolution in the use of gestures and by their role in generating new signs. In this paper, such an evolution is pointed out by the social activity of the students in a geometric context, where the main components are: the hands that shape geometrical figures, and the fingers that point to or trace geometrical entities (sides, angles, faces, vertices, etc.) related to the solution of the problem.

THE CASE OF GUSTAVO: SIGNS ARISING FROM GESTURES

In the following example, some students of the 8th grade in an Italian middle school are working in group to solve a geometric problem. The task is to find the solid figure that fills in the 3D hole obtained if two congruent regular squared-based pyramids are placed (on the same plane) with two sides of the bases touching each other (Fig. 1). The pupils are asked to not use any kind of concrete support, as drawings, paper and pencil, computers, etc. On the contrary, they are required to get the solution ‘in their mind’, simply imagining and discussing together about it (the tetrahedron ABCD in Fig. 1). The teacher in the classroom observes the work without interfering if not necessary. The students work in a big group

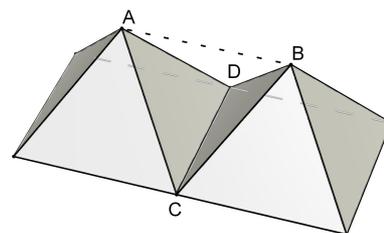


Figure 1

around some desks, in order to pay attention to what each other is doing or saying. During their curriculum they have already met regular tetrahedrons, as belonging to the family of pyramids (a regular pyramid with congruent faces and triangular basis).

The following analysis is based on the video recorded by a moving camera. In the course of the discussion, gestures are used in a massive and social way. Gestures are our data, and they are analysed to figure out the dynamics of the pupils' solution processes and communication acts. In particular, we will show the emergence of signs (arising from gestures) that can be seen, touched and manipulated (not simply imagined) by the students. These observations fit with the hypothesis stated above.

Since the beginning of the work, the actions of the group seem to occur principally around two pupils: Sara and Gustavo. In Fig. 2 Sara is the girl with long hair and eyeglasses on the right of the reader, and Gustavo is the boy on the left, with short hair. It is Gustavo who leads the game: he allows his mates to approach the solution and his gestures guide them and their gestures.



Figure 2

Three phases appear to characterise the dialogue of the students. First, the pupils need to re-construct the geometry of the given configuration. Gustavo performs a lot of gestures to imagine what he has in mind and makes it visible: fingers running along or pointing to Sara's hands open as they were the two close faces of the pyramids (Fig. 3a, 3b); hands closing the hole in the attempt of figuring out the unknown solid (Fig. 3c, 3d); fingers tracing sides of the solid itself (Fig. 3e, 3f).

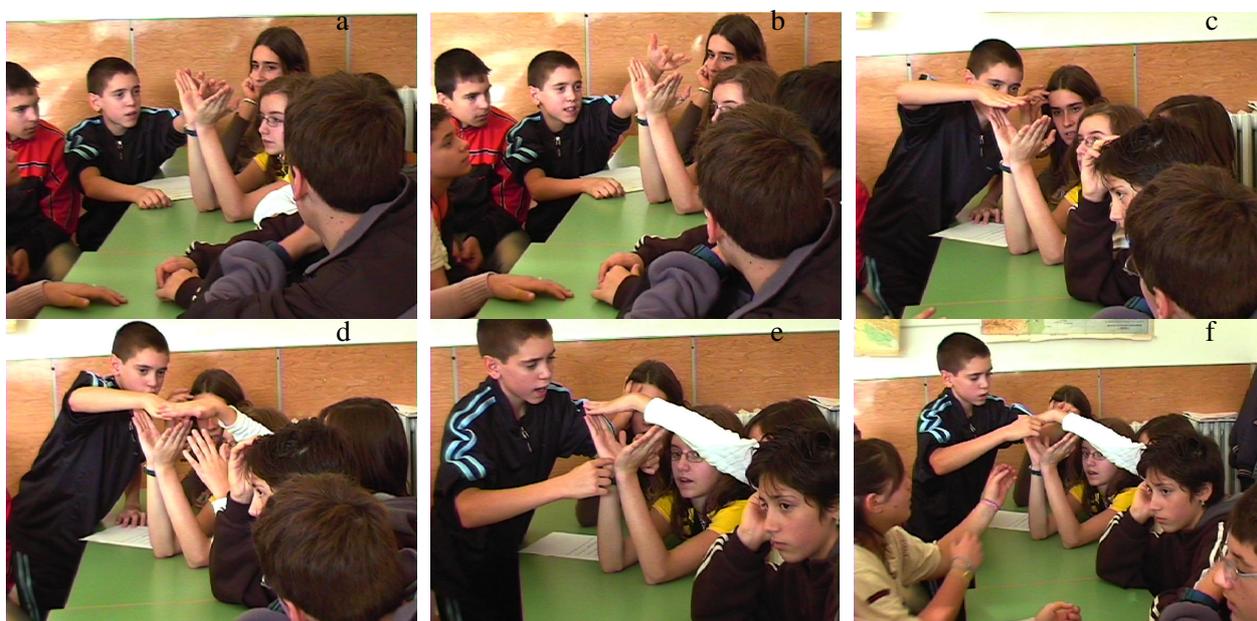


Figure 3

This is an *exploration* phase in which the students share a space of communication, that over the desks, where they can freely move, gesture, show. We can call it *gesture space*. To Gustavo, such a space represents something more: it is already a space of action and production, other than of communication (APC Space - Action,

Production and Communication - Arzarello, 2004). At the moment pupils need an investigation of this kind; spatio-motoric thinking is essential, since gestures are the only available means for them to understand the problem. Imagination alone is not sufficient to figure out the geometric entities, especially the one to be found.

Gustavo's gestures (performed in front of his body over the desks, Fig. 4) start to acquire a symbolic characterisation. Recalling Peirce's terminology, they are still *indexes* of figures (in that his hands in motion represent sides, faces, solids), but start becoming *symbols* when referring to the virtual world where these entities live and being used as really existing objects by Gustavo.



Figure 4

At this point, Gustavo has got the solid in his mind, as highlighted by his words:

Gustavo: It's a triangle, but with a thickness. It's a solid of pyramid.

A new phase begins here: in order to allow his mates to see the shape of the unknown solid, new signs are necessary, with a more concrete nature than the previous gestures. Their emergence is marked by a change of the gesture space, more visible. The (real) desks now represent the new space: in it, Gustavo's index finger traces some virtual segments to explain his solution to the others (Fig. 5).

Gustavo: It is in this way, in this way, and then in this way.



Figure 5

These segments are imaginary inscriptions, but used as if they were real, and so can be deleted (by hand in Fig. 6), just as geometric lines drawn on paper are erased by means of a rubber (the term inscription comes from Sfard & McClain, 2002).



Figure 6

This is a *conjecture* phase, but the explanation is not yet satisfactory to the group. The dialogue goes on among the pupils with an exchange of some sentences:

- Gustavo: It is made of two triangles with the bases below, and two triangles with the bases above.
- Sara: But, it's a solid. It [the problem] tells a solid, one!
- Gustavo: Yeah, it is a solid, made of two triangles placed with the bases below, which are those starting in this way and going up, and two triangles with the bases above that are those going in this way [see Fig. 7]



Figure 7

- Lucy: And how does it call?
- Gustavo: I don't know, but that's it.
- Sara: No, no, because above they have skew sides.
- Gustavo: No, it doesn't matter!

Let us observe that along the discussion, a conflict appears into the whole group: the opposite behaviour of the triangles above and below. Gustavo's metaphor on such triangles (expressed both in his words and gestures) is not useful to accept the shape of the searched solid. Hands are not enough. To overcome the obstacle, something different is to be used: something really existing, that can be effectively seen, touched, manipulated. Again, the emergence of new signs is strong. At this point a last phase of *production* begins: a tool enters the scene, as Gustavo claims:

- Gustavo: Guys, we got a tool! [he takes a piece of plasticine by the hands of a group mate]

Other than gestures, plasticine is stable, motionless and concrete: it is real, visible and manipulable. By plasticine, students are able to check their conjectures, to make the solution apparent, to create it. Figure 8 shows just some moments of this final phase.

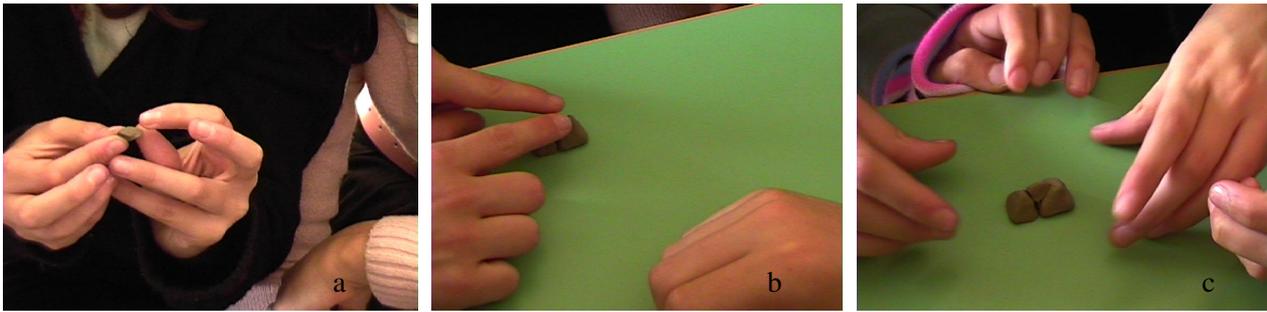


Figure 8

Somebody: It's a thing made in this way, it's strange [Fig. 8a]

Lucy: Try to put it [Fig. 8c]

Sara: But it is a pyramid with triangular basis.

All the geometric entities are visible; the tetrahedron itself is visible. In terms of Radford's words, students have objectified new knowledge: now they know that the solid solving the original task is a "pyramid with triangular basis".

SOME CONCLUSIONS

In the previous example we have described the activity of some students solving a 3D geometry problem. We have pointed out that their understanding grows up around the gestures of Gustavo, who is early able to imagine the solid solving the task, and in the same time has to explain it to the group mates. We have seen the way these gestures mark the birth of new perceivable signs: the virtual segments drawn by Gustavo's forefinger on the desks, and the solids shaped by the use of plasticine.

The relevant point of the activity is the evolution of gestures in generating signs. At first, Gustavo's gestures have an iconic function in that their shape resembles their referents (the geometric solids they express), but they become indexes (in the sense of Pierce) in his communicative attempt of transferring knowledge to the others. The indexical gestures acquire a symbolic function later, when they are used as existing objects of a virtual geometric world and in relation with the genuine geometric objects (e.g., think of the metaphor of the "two triangles with the bases below, and two triangles with the bases above"). This relation consists in a piece of theoretical knowledge. Particularly, when the students identify the unknown solid as a pyramid also their utterances have a two-fold nature: an indexical one and a symbolic one, in encoding information according to the theory at their disposal.

The most significant moment of the activity arises from the use of plasticine, here a tool expressing three different functions. First, plasticine has an iconic function for Gustavo, who wants to show the solution of the task, as well as his mates want to see it. Secondly, it is an index in being manipulated and seen; in these terms, it is a sign useful to make sense of the solution. But it also has the germs of a symbolic function in itself, because it is shaped in a theoretical knowledge, still vague if only supported by the previous gestures and metaphors. Gustavo and the group need to understand those relations. The discovery of the solid as a pyramid happens at this point.

We can interpret this last moment using the theory of instrumentation of Rabardel (1995) within a fresh viewpoint. According to Rabardel, an artefact becomes an instrument in the hands of a subject, by activating different schemes of use; in these terms, the instrument prompts the genesis of meanings. To our students plasticine is already an instrument: they are able to use it, and they have acquired the corresponding instrumentations. As a consequence, the role of plasticine is different from that of an artefact becoming an instrument; in a certain way, the situation is exactly opposite. Our students (especially Gustavo) take an instrument they have and know, with its schemes of use, and, through their perceptuo-motor information about it, they add new schemes of use in order to solve the problem. According to our wider frame developed for the Research Forum (see Arzarello *et al.*, 2005), the whole dynamics can be identified as a *SPO* (Serial Process of Objectification) in that the situation evolves by the successive production of signs (through gestures and words, and schemes of use).

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