

EXPLORING STRATEGIES USED BY GRADE 1 TO 3 CHILDREN THROUGH VISUAL PROMPTS, SYMBOLS AND WORDED PROBLEMS: A CASE FOR A LEARNING PATHWAY FOR NUMBER

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This study investigated how grade 1 to 3 children in South Africa learn early number concepts. A framework was developed and used to assess the children's level of understanding and used to analyse their strategies in solving number problems. Three schools were included and involved 222, 257 and 240 students in grade 1- 3 respectively. The children's level of understanding was assessed through the use of four tests. An analysis of performance, misconceptions and errors made by the learners in each grade was achieved through an in-depth analysis of 48 learners. The results suggested that the majority of learners were unable to solve straight calculations, employed the strategy counting all and counting on, while none engaged in formal or innovative methods. There is no progression in terms of conceptual mathematical development across the Foundation Phase.

INTRODUCTION

This study is located in a three year research and development project where the development of a learning pathway for number is the primary objective, but testing learners and classroom observations are strategies used to provide baseline information to measure the success of the project over three years and to feed into the development of the learning pathway. A detailed quantitative and qualitative analysis of the first round of baseline testing will be presented, a comparative qualitative analysis with the second round of testing, as well as suggestions on how these results motivate for the development of this learning pathway for number.

The development of a learning pathway for number in the early grades of the South African primary school (Foundation Phase) is intended as a mathematical guide for planning instructional sequences. The learning pathway for number is research-based and highlights the main features of children's early number development and describes how number knowledge, number sense, mental and written calculation, estimation and algorithms are developed and relate to each other within and across the Foundation Phase. Concepts and the number range for each grade in this framework are sequenced progressively as the 'stepping stones' that learners will pass on their way to reaching the Mathematics Assessment Standards related to number in the early primary grades. The learning pathway for number highlights the

cognitive and didactic continuum of number development across four stages, which give an overview of number development and details the progress that most learners will make on the route to being competent with numbers in different contexts and with different kinds of calculations.

THEORETICAL FRAMEWORK

A survey of international literature on children's numeracy (Kühne, 2004) shows a number of interventions and the research associated with this (in the Netherlands, Australia, the USA and the UK in particular – see Van den Heuvel-Panhuizen (1999), Bobis & Gould (2000), Clarkson (2000), Gould & Wright (2000), Carpenter, Fennema & Franke (1996) and Carpenter et al. (1999). All of these approaches add to our understanding of the development of number in the early years, and all, in some way, point to the need for a learning trajectory for number. This project needs to provide evidence of the efficacy of this pathway on teacher development and learner performance. To do this a baseline study, which measures the performance of foundation phase learners was mooted, hence the basis and the focus for this investigation. The aim of the project is to show a productive, efficient and sustainable way to lift learner's performance in mathematics.

The project design is located within what might broadly be described as “design research” (Brown, 1992). As Cobb et al. (2003) suggests, an important category of design research “seeks to develop an innovative intervention and an underlying theory that constitutes its rationale” (pg 2). The present project builds on a tradition that has already developed in relation to one-on-one teaching sessions, classroom design experiments, teacher development experiments and school restructuring experiments, to elaborate design research in relation to broad instructional tools such as the learning pathway for number described above. The framework draws on Steffe's (1992, 2000) scheme model and on the idea of emergent counting (Wright, 1998). The following framework is used to describe a trajectory for learning and how learners learn, understand and solve number problems.

STAGE 1	STAGE 2	STAGE 3	STAGE 4
Pre-school – Grade R	Grade R – Grade 1	Grade 1 – Grade 2	Grade 3 – Grade 4
Emergent Numeracy	Learning to count-and-calculate Integrated Mental and Written (operations up to 10)	Calculation by structuring Integrated Mental and Written (operations up to 20 and beyond)	Formal Calculation Mental Written (Operations up to 100 and beyond)

Levels within the stages: describe solution strategies

<p>Counting-and-Calculating Strategy Integrated mental and written Level 1 Count all in ones Count up to Take away Count down Level 2 <u>Stringing</u> Numbers objects in a counting row – operations along a counting row: beads, lines and tapes Count on forward/ backwards from first then biggest number Groups to count <u>Splitting</u> Breaking down and building up numbers Models Models ‘of ‘problems Line Group Combination</p>	<p>Calculation by Structuring Strategy Integrated mental and written Level 3 Contextualise numbers and number relationships in different situations <u>Stringing</u> Leaving first number intact and splitting second number <u>Splitting</u> Breaking down and building up numbers Doubling Halving <u>Varying</u> Range of ‘smart’ strategies Models Line Group Combination</p>	<p>Formal Operations Strategy Mental Written Level 4 Transfer knowledge of single digit no’s to 2 and 3-digit numbers Use decades as pivot points <u>Stringing</u> Uses decades as pivot points <u>Splitting</u> Decimal structure – both numbers to be added can be split into tens and ones. Jumps of 10 Jumps via 10 <u>Varying</u> Use of memorised facts, relations between numbers and properties of operations Rounding Off Compensating ‘Easy’ numbers Models Abstract Representations</p>
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TEST DEVELOPMENT

We consulted the relevant exit Assessment Standards (Benchmarked statements) of the Revised National Curriculum Statement (RNCS) for each grade and extracted the skills and knowledge components. A framework (table to be shown at the conference) was developed, which listed the generic skills relating to learning number. These skills remained constant across the grades, the range shows progression. A range of test items from various sources was selected and developed and the skills-framework was used to adapt these items into grade-specific test questions, ensuring that the ranges in the test items corresponded with that of the RNCS, thus ensuring item validity.

Some of the items lent themselves to presentation in three different ways. These were: contextual problems with graphic prompts (referred to as visual in the analysis), number sentences and sequences (referred to as symbols in the analysis), and the contextual question stated in words without graphic prompts (referred to as worded problems in analysis). Only the first six items of these three tests (called test A, B and C) are the same questions posed in different ways and using the same number range. It was intended that these items would give further indication as to possible barriers that might exist for learners when answering test questions. A fourth test (test D) was developed which contained items intended to test number recognition on number grids and in number sequences.

A test administrator’s guide (for the Grade One tests only) was developed as an example to demonstrate the methods of asking the questions. Experienced test administrators received additional training a week before the commencement of

testing. Test administration allocation of work was as follows: Grades 1 and 2, three test administrators per class and Grade 3, two test administrators per class.

DATA COLLECTION

The tests were piloted at a Primary School in the city. Three experienced test administrators were trained to present the tests to 20 grade 1 learners, 20 grade 2 learners and 20 grade 3 learners in three different classrooms. In grade 1 the test administrator read each item (question) in each test to the learners. The item (question) was only repeated once. In grade 2 and 3 the test administrator explained briefly the instructions for each test and not per item (question). At the end of the testing process, the Principal, the research supervisor, the test administrators, the grade 1, 2 and 3 teachers, as well as the Foundation Phase remedial teacher at the school discussed the rationale for testing at this level, the test instruments and the testing process. All the learners from two classes in each grade level in the three project schools were tested. The same procedure was used as described in the pilot implementation. The only change in terms of implementation was in grade 2 and grade 3 where each item was read twice to the learners in isiXhosa.

DATA ANALYSIS

A quantitative analysis involved an item analysis in the four tests per grade and a comparative analysis of the first six questions in test A, B and C highlighting scores for visual prompts (pictures), symbol (using +, - and = signs) and worded problems. A qualitative analysis involved the responses (strategies used) of 48 learners. Their methods and strategies, errors and misconceptions and their solutions were captured. However, for this study only the grade 3 results will be presented.

Quantitative Analysis

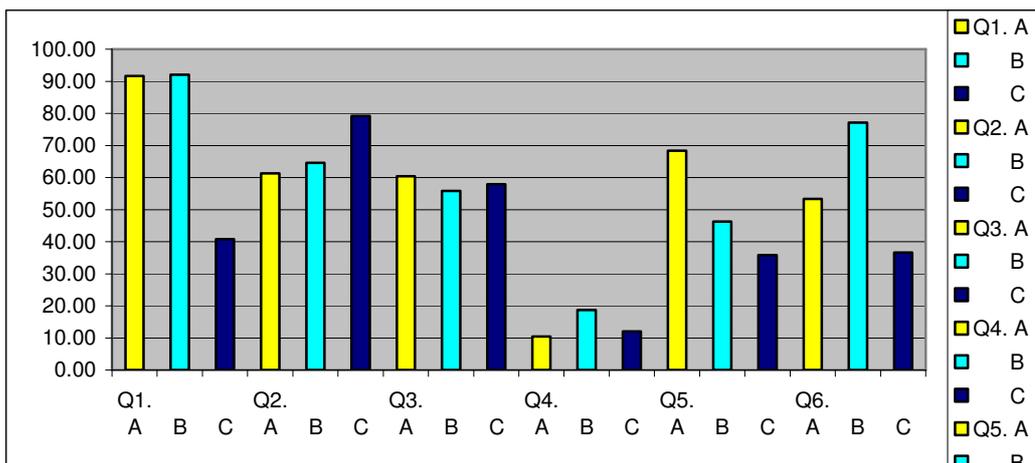
Grade 3

In test A, 91.6% and 68.3% were able to answer the questions on “how many”, which involved counting the number of turtles and counting with the use of plus and = signs. The performance on these items was also amongst the best in the four tests. The fact that 9.4% were unable to count the number of turtles is a cause of concern, especially for grade 3 learners. Counting backwards in twenty-five’s also proved to be beyond the capabilities of this group since they scored 0.8%. Counting in fives resulted in a score of 53.3% compared with the grade 2’s 32.3%. In the division question 62% of the learners were able to solve the problem based on ‘sharing’. The learners performed similarly to the grade 2’s with scores of 61.2% and 60% in the subtraction problems. The learners were unable to give a mid-number while counting in 50’s and scored 10.8% on this item only slightly better than the grade 2 learners.

In Test B only 92% of the learners could add single plus double digit numbers. The subtraction items yielded scores of 64.5% and 55.8% similar to the scores in test A. Adding in fives with plus signs between the numbers proved to be better than counting in fives without the signs. For the place holder type items the performance was only slightly better than the grade 2 learners with scores of 32%, 15% and 11.2% compared to less than 6% attained by the grade 2 learners on similar items.

In test C, the worded problems were also poorly done with the best performance, strangely enough, in a subtraction of double-digit numbers, where 79.1% achieved the correct answer. This was followed by a second best performance of 57.9% also in a subtraction item. In a ‘how many’ problem the grade 3 learners achieved only slightly better than the grade 2 learners with scores of 40.8% and in a ‘spending money’ problem they achieved a lower score of 13.7%. A low score of 2% was attained for a repeated addition problem.

In test D only 20% were able to fill in the missing numbers in a full grid of numbers. Again they performed slightly better than the grade 2 learners. The best performance of only 37% and 35.8% was achieved in ‘finding a number written in words in a part-grid’. This achievement was similar to grade 2 learners. The learners struggled with this number recognition test with the lowest performance of 4.1% in the item ‘find a 3-digit ‘between’ number written in words in a part grid’. The performance on ‘before’ and ‘after’ numbers was only slightly better with scores of 5% and 15% respectively. The graph below shows how grade 3 learners performed on the first six questions in test A, B and C. The graph compares the performance in visual, symbol and context type problems where the numbers were the same for each type.



Scores on first six questions in test A,B and C

Qualitative Analysis

The analysis of the learner responses showed that the strategies used for solving problems from grade 1 to grade 3 did not change, showing very little progression across the grades. Grade 3 learners in particular used counting strategies (count all and count on). There was no evidence of a calculation by structuring, for example grouping or breaking up numbers. The vast majority of learners simply wrote down the answers. Field-staff reported that fingers and in some cases even toes (some learners removed their shoes and some did not have shoes) were used to aid counting. The following represents a few grade 3 learner-responses, from one of the classes at a school, to some of the test items (A, B – indicates the group at the school, while 1,2,3 – indicates the learner number).

Test A

- A2 represented the problem in the answer block with tally marks:
- IIIIIIIII + IIIIIII = 19 for question 1. (correct response). The learners appeared to need visual prompts in order to count-calculate the answer.
- A28 drew 19 tally marks and cancelled 10 = 9 for question 2 (correct response) to represent subtraction
- A39 and other learners consistently wrote the numbers either upside down or inversely.
- A2 wrote $25 + 25+25+25+25+25$ correctly, but did not give an answer. Also used groups of 25 tally marks to assist, but could not get to the answer for question 4.
- A2 counted in 5's by adding the top, middle and bottom rows: $30+20+20 = 70$ for question 6. A strategy that worked well. Quite a few learners wrote $6 + 20+20$, they forgot to multiply the 6 by 5. Others simply added the number of fives and got 14, but forgot to multiply by 5.
- A7 simply counted the bags and not the kg's in the bag in question 7. A18 wrote 30050 instead of 350. This kind of response was quite common, even at this grade level.
- A2 wrote the subtraction number sentence, but gave the wrong answer: $50-12 = 50$ for question 8. The majority only counted the visible objects and did not take into account the hidden ones.

Test B

- A2, A14 and others drew tally marks. Others used little circles to represent their counting. It appeared as if they counted all the marks or counted on from one of the given number in question 1.
- A20 drew 19 tally marks – 10 tally marks, then cancelled 10 tally marks to get an answer of 19 in question 2. A number of the learners ignored the minus sign and simply added the two numbers. The majority of learners used tally marks and circles to get the correct answer.
- A2, A8, A14, A20 and others, all used the groups of 25 tally marks or circles, but were unable to give an answer. The majority showed a strategy but failed to give a correct response in question 4.

Learners used tally marks or circles to represent the problem and to assist in their counting-and-calculating. As the numbers range increased, however, errors in counting became common.

Comparative Qualitative Analysis

The strategies used by the 48 randomly selected learners were used to compare their performance in March and November. Table 1 represents a sample of each learner's analysis for test A.

SOME OBSERVATIONS

In general the scores improve in each grade. There is one exception: Grade 3 Question (where the oranges are hidden in a repeated addition problem), where the grade 1's scored 15%, grade 2's scored 24% and the grade 3's only achieved 12%. For most of the questions it is true that the word problems are the most difficult type of questions and problems with visual prompts appeared to be the easiest. Some problems behave "strange". This could be a result of how the problems are designed. Another example shows that the addition word problem in grade 1 (Q1C) is more difficult than the subtraction problem in grade 1 (Q2C), because "crawl in" is more difficult to understand and to see the underlying operation than it is in the case of "eat". None of the learners displayed higher order thinking skills. There was no evidence of formal/flexible operations.

Table 1 - LEARNER CODE: Im2

ITEM	MARCH STRATEGY	SCORE	NOVEMBER STRATEGY	SCORE	COMMENTS
1	Used 8 + 11 tally marks	Correct	No strategy	Correct	Learner able to solve correctly without any strategy
2	Wrote 10 + 19, but 9 in the answer block	Correct	No strategy	Correct	
3	No answer	Wrong	No strategy	Correct	
4	No answer	Wrong	Used 6 groups of 25 tally marks each, counted wrongly	Wrong	Too many tally marks to count
5	No attempt	Wrong	No strategy	Correct	Learner able to solve correctly without any strategy
6	Wrote sum in answer block $30+20+20=70$	Correct	No strategy	Correct	
7	Incomplete answer	Wrong	No strategy	Wrong	Learner unable to solve problems. Only counted the visible parts to the question. Learner probably made a slip with the 7.
8	Wrote $50-12=50$	Wrong	Counted chocs that were visible	Wrong	
9	Wrote $78-6$ but no answer	Wrong	Counted beads that were visible	Wrong	
10	Counted in one's	Wrong	Wrote 735,785,799. did not count backwards in 25's	Wrong	
11	Wrote Γ50 the 7 faced the wrong way	Wrong	No strategy	Wrong	
12	4 groups of 4 circles and wrote 16	Wrong	No strategy	Correct	Learner able to solve correctly without any strategy
13	Wrote $4+8=11$	Wrong	No strategy	Wrong	Did not understand the question
14	Shaded 2 x R5 coins and 5 x R1 coins	Correct	Shaded 3 x R5 coins	Correct	Different, but correct responses
		4 correct		7 correct	Improved slightly

CONCLUDING REMARKS

The fact that the tests were conducted very early in the year (only three months into the year) some concepts may not have been taught, revised or consolidated by the teachers at the schools, hence the series of poor responses on a number of concepts or test items. This only improved slightly in November despite a year's teaching. Based on the 4 stages of development, the grade 3 learners are operating at grade 1 and early grade 2 stages. The insights gained from the testing will be used for the development phase in 2005.

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