MEANING CONSTRUCTION THROUGH SEMIOTIC MEANS: THE CASE OF THE VISUAL PYRAMID

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This paper presents some elements of our study on the construction of mathematical meanings in terms of development of semiotic systems (gestures, speech in oral and written form, drawings) in a Vygotskian framework with reference to cultural artefacts (Wartofsky). It concerns with a teaching experiment on perspective drawing at primary school (4th-5th grade classes). We analyse the appropriation of an element of the mathematical model of perspective drawing (visual pyramid) through the development of gestures, speech and drawings, starting from a concrete experience with a Dürer's glass to the interpretation of a new artefact as a concrete model of that mathematical object.

INTRODUCTION

In Bartolini et al. (in press), we presented the rationale, design and early findings of a teaching experiment, carried out with 4th – 5th graders, on the mathematical modeling of perspective drawing. It is well known that early theoreticians of perspective conceived "*painting as nothing more than intersection of the visual pyramid*" (Leon Battista Alberti, *De Pictura*, 1540). The appropriation of this conception is a good opportunity for pupils to deepen spatial experience and to construct the meaning of mathematical objects, such as pyramid and intersection, as abstract entities.

THEORETICAL FRAMEWORK

The study was carried out in a Vygotskian framework, which has been gradually enriched with contributions by other authors. In short, the theoretical framework is built around three different poles: (1) the cultural-historical pole, to describe the features of technical and psychological tools; (2) the didactic pole, to describe the way of designing, implementing and analysing processes of semiotic mediation; (3) the cognitive pole, to describe the process of internalisation of interpsychological activity, that creates the plane of individual consciousness (Bartolini et al., 1996, 1999). To deepen the study of relations between technical and psychological tools ("language, various systems for counting, mnemonic techniques, algebraic symbol systems, works of art, writing, schemes, diagrams, maps, and mechanical drawings, all sorts of conventional signs and so on", Vygotskij 1974 – p. 227), we adopted Wartofsy's distinction between primary, secondary and tertiary artefacts (Wartofsky,

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1979, p. 200 ff). *Primary artefacts* are "those directly used" and *secondary artefacts* are "those used in the preservation and transmission of the acquired skills or modes of action". Technical tools correspond to primary artefacts, whereas secondary artefacts are representations, i.e. "physical and perceptual embodiments of a mode of action", realized by different semiotic means (e.g. gesture, speech, drawing). *Tertiary artefacts* usually correspond to objects that are described by rules and conventions and are not strictly connected to practice.

In the quoted study (Bartolini et al., in press), two hypotheses were stated and tested: 1) the Hypothesis of Polysemy: "The intrinsic polysemy of the artefact supports the production of the polyphony of voices, in classroom activities"; 2) The Hypothesis of Embodiment: "The concreteness of the artefact fosters the production of gestures, linguistic expressions and drawings; they are elements of complex semiotic systems that evolve in time, using classroom activities. These systems support the transition to secondary and tertiary artefacts". This second hypothesis has been modified slightly taking into account the importance of drawings in our experiment in accordance with the entire Vygotskian tradition (e.g. Stetsenko, 1995). The crucial educational problem is to create conditions where the polysemy of artefacts, which is related to the polyphony of classroom activities, can be internalised by pupils. The aim of this report is to defend the following thesis, which links the two hypotheses in a functional way: *the parallel development of several semiotic systems (gestures, drawings, oral and written language) produces the internalisation of the polysemy; it is led by the teacher, who draws on the concreteness of the artefact (hypothesis of embodiment).* These semiotic systems allow pupils to construct (or to appropriate) the meaning of mathematical objects, starting from an exploratory activity concerning a primary artefact, then secondary artefacts, to the mathematical model. In our study, the mathematical model is a visual pyramid. The basic elements of this model are: the construction of the pyramid (monocular vision and base), the intersection between the pyramid and plane of painting, which gives the prospective image of the chosen object, the similitude of the images obtained by cutting the pyramid with planes parallel to painting. Due to limitations of space, in this paper we only consider the first element; the second one is analysed in (Bartolini & al., in press; Bartolini & Maschietto, in press). To discuss our thesis, we analyse several excerpts of protocols from different steps of the teaching experiment, presented in the following paragraph.

THE TEACHING EXPERIMENT

The teaching experiment, which was composed of several steps (Bartolini et al., in press; Maschietto et al., 2004), began at the end of the 4th grade course (steps 1 and 2 – May 2002) and constituted a part of the mathematical curriculum of the 5th grade course (from October 2002). In this paper, we consider the steps described below.

- Step 1. Exploration of a primary artefact, i.e. a Dürer's glass (Figure 2). This is the simplest perspectograph, composed of an eyehole and a transparent screen, where the artist traces the apparent contour of the object directly. A copy of a it was explored during a mathematical discussion (Bartolini, 1996). It is made of wood, plexiglas and metal; it has three eyeholes but only by looking through the one in the middle can you see the drawing superimposed on the skeleton of a cube inside (Figure 2). Pupils were asked to use it: each pupil was asked to look through the eyehole and compare the different images he/she could see through the different holes.

- Step 2. Pupils were asked to draw the artefact that was sitting on the teacher's desk.

- Step 3. Interpretation of secondary artefacts. During a mathematical discussion, pupils were asked to interpret some excerpts of texts drawn from ancient treatises on paintings (Piero della Francesca, L.B. Alberti). Among these excerpts, the sentence that introduced the first mathematical model for Dürer's glass, which was no longer available in the classroom, was "Thus painting will be nothing more than intersection of the visual pyramid" by L.B.Alberti (*De Pictura*, 1540).

- Step 8. Individual text on Alberti's visual pyramid: Alberti's sentence on the visual pyramid was proposed for an individual task three months after step 3.

- Step 9. Exploration and use of a new artefact (Figure 1): a model of the visual pyramid was introduced into the classroom. This model is made of wood (poles and glass with hole) and threads forming the edges of a pyramid. Each thread passes through a horizontal plane with holes and is taut: one end is fixed inside the hole and the other is attached to a weight. Another pole with some small holes in is present. A discussion followed.

Figure 1: model

In this paper we analyse the construction of the visual pyramid (vertex, faces and edges) through the development of gestures, speech (in oral or written form) and drawings. At the end, we show how a new artefact, very similar to the explored Dürer's glass in material and design, was instead interpreted by the pupils, as a concrete model of an abstract mathematical object, which highlights its polysemy.

THE ANALYSIS OF PROTOCOLS

The analysis is based on several kinds of data that were collected: individual protocols (texts, drawings); audio-recordings (and sometimes video-recordings) of classroom activities; photos of the pupils at work; and the teacher's and observer's notes.

Speech and gestures (step 1 – Discussion on Dürer's glass)

The focus on the functioning of the model evoked previous experiences with other objects with a monocular vision (e.g. camera, video camera) and allowed the pupils to pay attention to the importance of the point of view, which was the position that implies coincidence between the drawing and the cubic frame.

AleB I think there is only one hole as you can see the perspective better with one eye. If you look at it with two eyes it is different.

Fede Because if I keep my eyes open, and now I place my fist in front of the edge of this ... of this machine, I get confused between my hand and the wooden stick and I don't understand anything. If I place my fist against the edge and close one eye I can see both perfectly.

Figure 2: exploration of artefact

Figure 3: Ange's drawing

Analysis of the discussion shows that the exploration of the perspectograph fosters the production of linguistic expressions and gestures, such as: closed eye/open eye, and index finger (Figure 2) to explore the relationships between the drawing on the glass and a real object (cubic skeleton). They contribute to the construction of simple operational schemes of the artefact. Since they develop in the context of a mathematical discussion (that is, a situation of interaction), they can also be considered the germs of a secondary artefact aimed to the transmission of modes of action.

Drawings (step 2 – Individual drawings of the perspectograph)

In Figure 3, the evoked modes of action are: pupil position with respect to the three holes indicates the correct choice of central ocular (superimposition of the drawing on the glass over the cubic skeleton); the position of the pupil's arm is similar to the position in Figure 2. This drawing also shows the result of the seeing action through the chosen eyehole: the contour of the red cubic skeleton is black, which corresponds to the real view through the central eyehole. Figure 4 contains the identified elements in a different way: the choice of the eyehole (and so the singleness of the point of view) is demonstrated by the sentence "the bigger hole (...)" and corresponding drawing; the result of the seeing action is drawn with a comment indicating the mode of action ("If I look through the eyehole, I see the cube fits together with the contour"). Figure 5 shows the use of a single eye, with the choice of the central hole. Some of these drawings can be interpreted as secondary artefacts, as they present modes of action of the perspectograph. They also contain gestures that appeared in the previous activity.

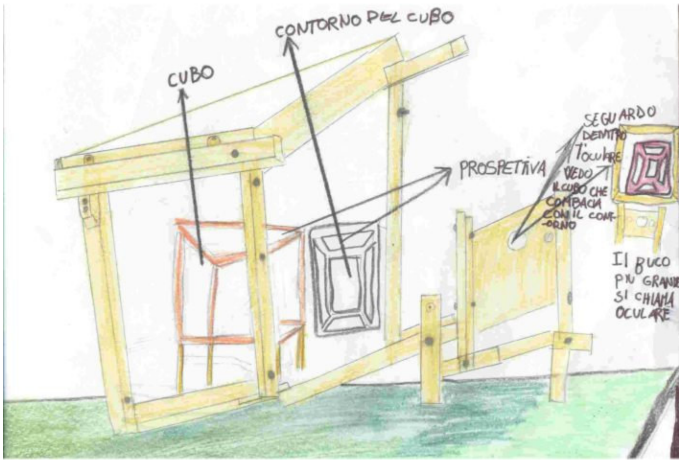


Figure 4: Ales's drawing

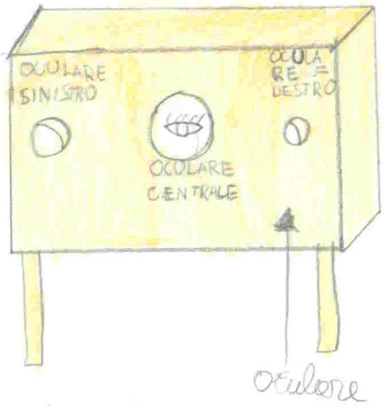


Figure 5: Carl's drawing

Speech and gestures (step 3 – Discussion on Alberti's sentence)

AleB — If the base is triangular it has 4 [faces], if the base is square it necessarily has 5. It depends on the base. The one we are talking about has either a square or a rectangular base, because we imagine a painting or a piece of glass and the point of the triangles reaches the eye.

Fede — Yes, but Leon Battista Alberti's is not a real solid, it's an imaginary solid which takes shape while you're looking at it. We can't see it we can see it only when we think of it, if we want to see it. For example we can see it now because we have just read it.

Assia — Of course it's imaginary, otherwise it would harm you and then it wouldn't even allow you to see.

Voices — Can you imagine a solid getting into your eye!

[*Many gestures, funny ones as well! A moment of confusion and jokes about the visual pyramid with participation of the entire class*].

Pupils' statements referred to two elements of the visual pyramid: the arbitrary base and the vertex entering an eye. So, the single eye used for the perspectograph takes on the new role of vertex (into play between real and imaginary object).

Drawings and text (step 8 – Individual comments on Alberti's sentence)

Individual protocols on Alberti's sentence seem to reveal that the visual pyramid had been internalised. In pupils' drawings, gestures that mimed planes and lines during the discussion of step 3 became very precise signs. They bore traces of the single eye arising from both the use of the perspectograph and the appropriation of Alberti's sentence. For the first case, the traces are: looking with one eye closed and the other open (Figures 6 and 7), the gesture of closing one eye was explicitly evoked by the drawing and accompanying comment (Figure 9, where the pupil raised her hand to close her eye). In his drawing (Figure 7), Giac gave an explicit reference to his experience with the real instrument, because he labelled the open eye as an "eyehole". In the second case, the eye was the vertex of the pyramid (Figure 6, 7, 8 and 9), which was a visual one in the pupils' comments; so that eye was the only one considered.

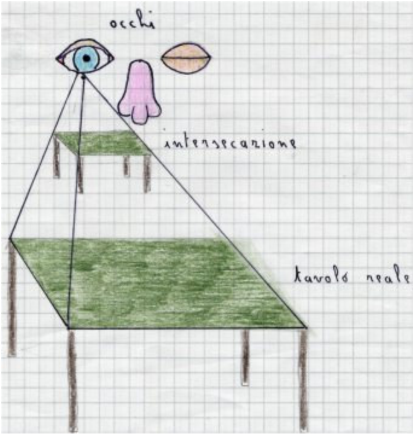


Figure 6: AleB's drawing

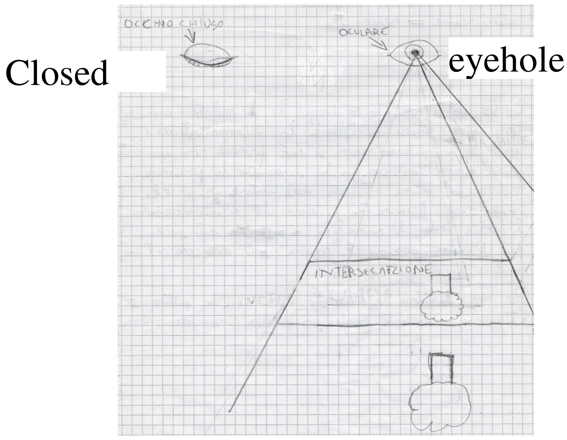
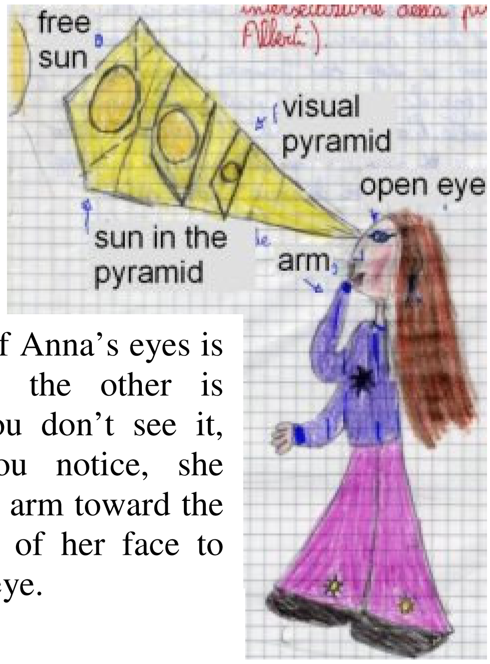


Figure 7: Giac's drawing

Figure 8: Elis's drawing



(...) One of Anna's eyes is open and the other is closed; you don't see it, but if you notice, she moves her arm toward the other side of her face to close her eye.

Figure 9: Ange's drawing and text

Speech and gestures (step 9 – Discussion on a model of the visual pyramid)

At the beginning, the teacher encouraged the pupils to explore the artefact (Figure 1).

AleB	All [the threads] go to the same point.
Teacher	Exactly. Very good, Ale. All [the threads] go to the same point, which must be the vertex ... of what?
Voices	Of the pyramid [*very quietly*] [...]
Voices	To make the intersection.
Teacher	(...) And so, where must the upper vertex of the pyramid be?

[*Silence among the pupils*]

Daniele	The vertex of the pyramid must be [*his arm reaches the glass with the hole*] on the other side of the visual pyramid, so more or less here [*his hand indicates a space beyond the glass*] ... here [...]
Maru	Franca, in this [*he touches the pole with the holes*] without [*he indicates the glass with the hole*] you can see it from different points of ... [...]

Maru	Of view
Elis	Before, we said that it [*upper vertex*] was here (Figure 10) since it was a point [...]
Voices	Imaginary
Marc	The pyramid is ... is continued by the eye when you see through the eyehole, but it is imaginary, it is not seen.
Ale	It is as if those threads [*left hand raised to his eye*] formed your imaginary pyramid.
Anna	To understand the pyramid.
Ale	The one you would make with your eye.

Figure 10: Elis's gesture

Figure 11: seeing with the model

Although the exploration of the object was physical and accompanied by new gestures (referring to edges, faces and vertices of that pyramid built by threads), the pupils seemed to appropriate it as a secondary artefact. They considered it as a model (statements by Marc, Anna and Ale). Gestures related to the process of appropriation of the artefact (for instance in Figure 10, Elis pulled the "virtual" threads in the air, beyond the glass with the hole), determined the constitution of modes of action (Figure 11), which was different from the perspectograph one (Figure 2). In Figure 11 pupil did not put his eye near the hole (as in Figure 2), but he placed it a few centimetres away, where the vertex of the pyramid must be. During this process, the pupils made gestures that were present in the previous activities: for example, during his speech ("It is as if those threads (left hand raised to his eye) formed your imaginary pyramid"), Ale raised his right hand to his eye to indicate the position of the vertex. After these statements, another pupil, Maru, suggested some changes to the model to the teacher: by evoking the perspectograph, he suggested inserting a new glass to obtain a different intersection. This discussion presents two generalisations: 1) changes to the point of view (vertex) and pyramid base (choice of different polygons); 2) intersection between the glass and threads (the glass used in different positions). These facts reinforce the role of the model of this artefact for pupils.

CONCLUDING REMARKS

In this paper, we have considered the relationship between the Hypothesis of Embodiment and the Hypothesis of Polysemy stated in Bartolini et al. (in press). The internalisation of the polysemy is achieved through the parallel development of several semiotic systems. It starts from activities concerning primary artefacts, then secondary ones to mathematical models. During this long process, teacher mediation plays a crucial role. The analysis of pupils' protocols highlights not only the parallel development of different semiotic systems (gestures, speech in oral and written form, drawings), but also their mutual enrichment as time goes by. They are not independent of one another, but complementary. For instance, the text is not just a simple comment on a drawing (and vice versa), but text and drawing contribute to meaning construction. In a lot of protocols, the drawings seem richer than the accompanying written text when present. This fact is consistent with Stetsenko's claim (1995) of the importance of drawing in pupils' evolution. Gestures in the air simulated points (e.g. the vertex), straight lines (e.g. the threads extended in visual rays), planes (e.g. the faces of the pyramid are mimed by hands touching two threads simultaneously). They were connected on one hand to pupils' concrete experiences, on the other hand to the abstract and general mathematical model of the visual pyramid (for other details, Bartolini & Maschietto, in press).

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