SPREADSHEETS, PEDAGOGIC STRATEGIES AND THE EVOLUTION OF MEANING FOR VARIABLE

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We report on one aspect of a longitudinal study which seeks insight into the ways in which spreadsheet experience and teachers' pedagogic strategies shape pupils' construction of meaning for algebra. Using data from stimulated recall interviews we analyse the evolution of meaning for variable through the mediation of the variable cell and the mediation of naming a column. We discuss metaphors of change and dragging, together with the process of naming.

BACKGROUND

Research suggests that spreadsheets can support pupils in developing an understanding of variables. In a longitudinal study of two groups of 10-11 year old pupils working on traditional problems, Sutherland and Rojano (1993) conclude that 'a spreadsheet helps pupils explore, express and formalise their informal ideas' (p.380), moving from thinking with specific numbers to symbolising a general rule. Moreover, it is claimed that spreadsheet notation 'can ultimately be used as cognitive support for introducing and for sustaining the more traditional discourse of school algebra' (Kieran, 1996, p.275). Dettori et al. (1995) discuss the benefits and limitations of such use of spreadsheets, suggesting their value in supporting pupils' understanding of what it means to solve an equation, for example, and their limitation in the formal manipulation of relations.

Sutherland (1995) found that low achieving 14-15 year olds, who had worked on a unit which required them to write an algebraic version of a spreadsheet formula, were able to use their knowledge in a paper and pencil test. One pupil drew a spreadsheet on paper and was able to represent the relationships using letters when subsequently interviewed. Sutherland concluded that 'the spreadsheet symbol and the algebra symbol came to represent "any number" for the pupils' (p.285). Mariotti and Cerulli (2001) similarly report that in a paper and pencil environment, pupils used signs derived from their symbolic manipulator 'L'Algebrista.' Several researchers point to the important role of the teacher in guiding pupils' construction of meaning when working with technological tools (for example Dettori et al, 1995).

The framework of semiotic mediation is useful for interpreting the role of technological tools in a didactic situation. Rooted in the work of Vygotsky, semiotic mediation refers to the mediating function of signs and tools on the learners' construction of meaning. Mariotti (2002) identifies two levels of semiotic mediation.

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At the first level, meanings emerge directly from pupils' activity, and hence the tool functions as a semiotic mediator. At the second level

> '(Since) the *mathematical* meaning incorporated in the artifact may remain inaccessible to the user … evolution is achieved by means of social construction in the classroom, under the guidance of the teacher' (Mariotti, 2002, p.708)

Spreadsheets offer access to the meaning of algebra through the use of formulae and graphing and specifically to the meaning of variable through the notion of a 'variable cell' and 'variable column' (Haspekian, 2003). In this paper we consider how a teacher guides the evolution of meaning for variable, focusing in particular on guiding meaning for variable in paper and pencil activity.

This study builds upon the Purposeful Algebraic Activity project[1] which aimed to explore the potential of spreadsheets as tools in the introduction to algebra and algebraic thinking. The project involved the design and implementation of a spreadsheet-based teaching programme with five Year 7 classes (aged 11-12). Two of the tasks in particular involved moving away from the spreadsheet and making some links to standard notation. Of relevance here is the finding that different tasks offered different opportunities for pupils to construct meaning for variable (Ainley, Bills and Wilson, 2004) but that not all pupils seemed to construct this meaning for spreadsheet notation. We have also found that spreadsheet affordances can support pupils' paper and pencil generalising (Wilson, Ainley and Bills, 2004).

These emerging findings have informed the development of this study which focuses more closely on the guidance of the teacher. It seeks insight into the ways in which spreadsheet experience and teachers' pedagogic strategies mediate pupils' construction of meaning for algebra. As part of the study, one of the classes who participated in the teaching programme was traced into Year 8 (aged 12-13). The class of high attainers was taught by Judith, an experienced mathematics teacher who was familiar with the content of the teaching programme (having taught it to other classes in Year 7). During this year, the class participated in follow up work, planned in collaboration with Judith and driven by the demands of Year 8 curriculum together with the affordances of the spreadsheet. Judith also taught some additional algebra lessons which focused on simplifying expressions and solving equations.

PEDAGOGIC STRATEGIES

Judith employed specific pedagogic strategies with the aim of guiding the evolution of meanings for algebra and making links to the paper and pencil activities in the curriculum. Such strategies were employed at various times during the year but mainly fell into four series of lessons within algebra units of work. An overview of these lessons is given below. Some lessons took place in a computer room, others took place in the classroom, often utilising the projected image of the spreadsheet.

[1] Both the Purposeful Algebraic Activity project and this study are funded by the Economic and Social Research Council

Series of lessons	Pedagogic strategies
'Odd one out' lessons used the spreadsheet as an environment for generating equivalent expressions. Pupils were given the task of identifying which formula or expression was the odd one out and then were invited to make up their own odd one out games.	• Emphasising idea of the variable cell and the variable column • Making links between spreadsheet notation and standard notation
'Myphone' lessons built upon and extended work from Year 7. Pupils set up a spreadsheet to show the cost of calls under different tariffs and solved various problems on the spreadsheet and by solving equations.	• Naming a column on a spreadsheet • Making links between spreadsheet formulae, equations and graphs • Focusing on what it means to solve an equation
'Sum and product' lessons built upon and extended work from Year 7. Pupils used trial and improvement in both environments to solve quadratic equations, one esoteric and one about a sheep pen of a given area.	• Naming a column on a spreadsheet • Focusing on what it means to solve an equation
'Generalising' lessons involved writing spreadsheet and algebraic formulae to represent various relationships, and then solving problems using various methods.	• Writing formulae and using substitution through work on printed screen snaps • Using spreadsheet affordances - focus on calculations, use of notation, feedback • Considering different solution strategies

This paper focuses on three strategies which specifically relate to the meaning of variable: emphasising the idea of the variable cell and column; making links between spreadsheet notation and standard notation; and naming a column on a spreadsheet.

DATA COLLECTION

In each of the lessons, a range of data was collected including field notes, audio and video recordings of Judith's teaching, and video and screen recordings of the activity of a pair of pupils. Following each series of lessons, a small group of pupils was interviewed using the technique of stimulated recall (Calderhead, 1981). The pupils were invited to watch short video clips from the lessons, some with and some without sound, and asked questions such as: Can you remember what was happening here? What do you think Judith meant when she said …? Can you remember what you were thinking? One focus of the discussion was pupils' construction of meaning for variable. The discussions were characterised by the openness of the pupils who had known the researcher for over a year and who represented the range of attainment within the set. Although the pupils were not always confident that they could recall what they were thinking at the time of the lesson, the interviews provided some valuable insights into their interpretations of notation and pedagogic strategies.

Alongside the data from the lessons, pupils were also interviewed in pairs at the beginning and end of the year, although there is not space to refer to this data here. Judith was also interviewed at the end of the year. The data was semi-transcribed and transcripts were interwoven with non-verbal behaviour, any written work and interaction with the computer as appropriate. In our initial analysis, we outline the affordances offered by the spreadsheet and then consider the evolution of the meaning for variable within the field of experience of pupils' spreadsheet-based activity. We focus in particular on the mediation of the variable cell and of the process of naming a column in making links to standard algebraic notation.

SPREADSHEETS

The spreadsheet environment offers three important affordances related to variable which we consider in this paper: the variable cell, the variable column and the named column. Haspekian (2003) identifies four features of a 'variable cell,' such as A2. The first feature corresponds with the use of a letter to stand for a variable:

- 'an abstract, general reference: it represents the variable
- a particular concrete reference: it is here a number
- a geographic reference
- a material reference' (p.6)

Spreadsheets also have the facility for filling down a formula through a range of cells and generating a 'variable column' (Haspekian, 2003). A name for a column, or indeed a cell or row, can also be defined and that name used in a formula (the 'A' column is defined as 'n' in the example below). When the formula is filled down, each new formula then includes the same name. This facility has not been widely researched in terms of pupils' algebraic thinking.

	A	B
1	1	=2*A1+3
2	2	=2*A2+3
3	3	=2*A3+3
4	4	=2*A4+3

	A	B
1	1	=2*n+3
2	2	=2*n+3
3	3	=2*n+3
4	4	=2*n+3

EVOLUTION OF MEANING FOR VARIABLE

In the spreadsheet environment, the activity of writing formulae to solve problems involves using notation, such as A2, but pupils may or may not recognise that the notation represents each of the features of the variable cell, and in particular an 'abstract, general reference,' a variable (Haspekian, 2003). This meaning emerges from the pupils' activity and reflection guided by the teacher. In this case, Judith emphasised the idea of the variable cell and variable column, making links to the use of standard notation in the paper and pencil environment.

Mediation of the variable cell

In a number of the lessons in the teaching programme and then in the follow up work described earlier in the paper, the pupils had worked on various tasks which involved writing formulae and then changing the value in a cell. In the 'Odd one out' series of lessons, pupils were given various spreadsheet expressions such as '6*A2+12',

6*(A2+2), '2*(A2+2)+4*(A2+2)' and '5*(A2+1)+8' and were set the task of finding the odd one out by 'testing' various values in A2. Judith made links between the spreadsheet notation of A2 and standard algebraic notation which the pupils had already been introduced to ("denotes emphasis on the following syllable).

Judith	This A2 that we've been thinking about has been a "cell (hand gesture, box) on the spreadsheet that we were looking at last lesson (…) … Could I put any number into that A2 box? (substitutes 3 and then 7 for A2 in the formula written on the whiteboard) … When we can change a number … we're thinking to write maths down … what do we normally do? Yeah
Pupil	Put *x*
Judith	We usually put an *x*, a missing number, *x*. Just because we were thinking of this as a cell on the spreadsheet it's the "same as using algebra, it's the same as putting a missing number, which we usually put *x* … Could I use any other letter? Could I have put a *q* in there? …
Pupil	You can use any letter
Judith	You can use any letter. So instead of using A2 now we're gonna look at can you do it with any other letter, okay, because that letter just represents any number … So if I change all of these to *x*'s, does it still work? (erases each A2 from the formula on the whiteboard and replaces each with *x*)

Judith emphasised the idea of a variable cell, firstly using the image of the cell as a 'box,' literally replacing A2 with different numbers, and then making the link to standard notation. In their subsequent work on equivalent expressions using letters pupils could draw upon this image of variable. When asked in the stimulated recall interview about the values that A2 and *x* could take for expressions to be equivalent, Jason referred to the link that Judith had emphasised.

Researcher	Could it [*x*] be a decimal number? …
Jason	Yeah, it's the same as A2

Beatrice said that she thought that *x* meant 'any number' but she wasn't sure whether the expressions were equivalent if *x* was a decimal or a negative number. In contrast, when asked about similar work using A2 in her book, Beatrice felt strongly that A2 *could* be a decimal or a negative number in the equivalent expressions. This is an indication of the complexity of pupils' interpretations of notation.

In terms of the rationale for replacing A2 with *x* in the paper and pencil environment, Judith referred to the socio-mathematical norm of using a letter as 'what we normally do' and referred in later lessons to potential confusion if A2 was used on paper. Indeed, in one of the 'Odd one out' lessons, Erin had tried to multiply 4 by A2, and 2 by A2 on paper, but had written the products as 8A2 and 4A2 respectively. She commented on this possible source of confusion in the interview:

Erin	Because in the classroom they might think it's A times 2 … normally in algebra, when you're doubling something, you should put (.) say if it was *x*, it would be 2*x*. So they might think A is just another *x*

Other pupils, however, felt that using A2 on paper would not be confusing because in

standard algebra 'it's 2*a*' (rather than '*a*2') and 'algebra letters are never capitals.'

Analysis of the interview with Judith indicates that she sees the variable cell as important in mediating meaning for variable. Further, she pointed to the distinction between changing a number in a cell and filling down a series of calculations.

> Judith The "single cell … I think it reinforces the idea of a variable … having one number that can be changed for anything is slightly different than having a number, and an answer, and a "different number and an answer … I can't put my finger on "exactly what, but a few times I've got the impression from what the children have said. They don't "quite see that this calculation here is the same as the next one above but with a different number in your variable position, if you see what I mean. They think of it as a different position almost and therefore not the same, not quite the same variable

Judith's perception of the pupils' construction of meaning is insightful in two important and complementary ways. The variable cell offers the metaphor of kinaesthetic and visual *change*. Pupils *see* that a range of values can be entered into the 'material reference' (Haspekian, 2003) of the cell. They can also *see* that the formula which includes the variable cell *does not change*.

The variable column offers the metaphor of kinaesthetic and visual *dragging*. When a formula is filled down through dragging, pupils *see* a range of values as a list in the variable column. The formulae then include different variable cells: A2, A3, A4 etc. which, in Judith's words are 'not quite the same variable.' In terms of making links to standard algebraic notation, the inter-relation between this series A2, A3, A4 etc. and a single letter is more complex than for the variable cell. Yet an understanding of this relationship can potentially support pupils in their work with literal symbols through the metaphor of dragging. In the context of activity involving the variable column pupils tended to talk about 'the number that is in A2,' focusing on the 'particular concrete' (Haspekian, 2003) interpretation of A2. This is consistent with the fact that pupils tend not to change the number in A2 when they fill down.

Mediation of naming a column

In the 'Myphone' and 'Sum and product' lessons, Judith taught the class how to name a column on the spreadsheet. The aim was twofold: to make clearer the links between spreadsheet notation and standard algebraic notation; and to encourage pupils to see the notation as representing a variable. We do recognise however, as some pupils did, that there is no real reason to name a column on the spreadsheet.

> Judith [Discussing the formula =A2*0.16+15] We don't wanna call those cell A2, B2, er, A3, A4, A5 … We want to give them a letter like we would do in the classroom if we're gonna do that algebraically … Now instead of having to write down it's that cell times nought point one six plus fifteen, you can now say it's nought point one six times *x* if you've used *x* or nought point one six times *m* if you've used *m* and add fifteen

Our analysis suggests that naming a column mediated meaning in two ways. Naming a column involves highlighting the column and 'defining' a name. This engages pupils in the *naming* process and provides an image for variable as a range of numbers. The pupils' language reflects this active process, for example 'we had to minus the *a*, because we defined that as the *a* thing.' When asked to explain what *a* or *m* meant in a formula, pupils such as Julian clearly drew upon this image:

> Julian A2 is just that individual column (points to a single point), column, like cell thing (makes box shape with hands), the *a* is the actual whole thing column (moves hand up and down)

When writing a formula, rather than clicking on a cell and perhaps ignoring the notation, pupils type the name such as *a* or *m*, engaging directly with the notation.

As well as the image associated with *naming*, the named column (like the variable column) also offers the metaphor of *dragging*. Here, unlike the variable column, the links between notation are clearer, although there are different conventions in standard notation, such as omitting the * for multiply and writing 6*a*, for example, rather than *a*6. For these high attaining pupils the use of a single letter seemed to be a helpful bridge and the differences in convention were unproblematic.

Judith was also positive about naming a column, seeing it as a valuable strategy in making links to paper and pencil algebra and curriculum demands.

> Judith The naming of the column I think is successful in terms of them linking the algebra, certainly from my point of view … I think it helps them to see, instead of it's, well it's anything in that column, that it's "n, if you see what I mean … that column is a "variable column as opposed to just somewhere where you put a sum, if that makes sense

Judith also perceived that naming a column helped some pupils to identify the variable and to set out their work.

DISCUSSION

We have discussed the evolution of meaning for variable in one class of high attainers, drawing upon data in which pupils and their teacher were invited to discuss their learning and teaching. In particular, we attempted to analyse the mediation of the variable cell and the process of naming a column under the guidance of the teacher. The dynamic metaphors of *change* and *dragging* together with the process of *naming* appeared to support the evolution of meaning for variable.

Most pupils interpreted A2 as 'any number' in the context of work with variable cells, supporting the findings of Sutherland (1995). However, analysis of discussion around activity involving the variable column suggests that pupils' interpretations are context specific. We recognise the complexity of researching pupils' interpretations and the limitations of our analysis given that there was not space to refer to interview data and observations of pairs. Nonetheless, we suggest that naming a column can potentially support pupils in developing a clearer sense of the notation as a variable.

Whilst some pupils comment that 'when you do spreadsheets then you do algebra,'

others, perhaps with a narrower conception of what algebra is, say that 'it doesn't really feel like algebra on the spreadsheet.' Does this matter? Is it a good thing? Is it important to make links to paper and pencil activity? We would not want to suggest that spreadsheet activity is valuable only as a preliminary to introducing the traditional discourse of algebra, but we do suggest that rich spreadsheet activity can be invaluable in supporting pupils' construction of meaning for algebra. Our data points to the important role of the teacher in guiding this evolution of meaning, as illustrated in the words of Erin talking about spreadsheets and algebra:

> Erin I see links between them when she [Judith] talks about links between them but when they're like separate then I think they're separate (laughs)

In this ongoing research we are also analysing the mediation of writing formulae and graphing activity in the evolution of meaning for algebra.

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