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NECESSARY CONDITIONS FOR A SUPERDIFFERENTIABLE SUPERCURVE TO BE A WEAK MINIMUM RELATIVE TO TWO SUB-SUPERMANIFOLDS

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Abstract. Let L defines a regular problem in the calculus of variations on supermanifolds. The necessary conditions for a piecewise superdifferentiable supercurve C in sense of Rogers to be a weak local minimum relative to two sub-supermanifolds are given.

Let V be a supervector space [3], V^* be the dual supervector space [5], M be a supermanifold in the sense of Rogers [7] and $T(M)$ be the tangent superspace or superbundle [5] over M.

Let us consider only algebras over the real numbers. For each positive integer L, B_L [7] will denote the Grassmannian algebra over the real numbers with generators 1^L, $\beta_1^L, \ldots, \beta_L^L$ and relations

$$1^L \cdot \beta_i^L = \beta_i^L \cdot 1^L = \beta_i^L \quad i = 1, \ldots, L\,,$$
$$\beta_i^L \cdot \beta_j^L = -\beta_j^L \cdot \beta_i^L \quad i, j = 1, \ldots, L\,.$$

B_L is a graded algebra [8] and can be written as a direct sum [7]

$$B_L = (B_L)_0 \oplus (B_L)_1$$

where $(B_L)_0$ and $(B_L)_1$ are the even and the odd parts of (B_L) respectively. We consider the (m,n)-dimensional supereuclidean space $B_L^{m,n} = (B_L)_0^m \oplus (B_L)_1^n$ [7] with $L > n$. Let M_L denote (following Kostant [6]) the set of finite sequences of positive integers $\mu = (\mu_1, \ldots, \mu_k)$ with $1 \leq \mu_1 < \cdots < \mu_k \leq L$. M_L includes also the sequence with no elements, which is denoted by ϕ. As it follows from [6] for each μ in M_L

$$\beta_\mu^{(L)} = \beta_{\mu_1}^{(L)} \cdots \beta_{\mu_k}^{(L)}, \quad k = 1, \ldots, L$$