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CLASSICAL AND QUANTUM COLLECTIVE DYNAMICS OF DEFORMABLE OBJECTS. SYMMETRY AND INTEGRABILITY PROBLEMS

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Abstract. Discussed is affine model of collective degrees of freedom of multi-particle systems or continuous media. The novelty of our approach is that it is not only kinematics, i.e. geometry of degrees of freedom, that is invariant under affine group, but rather we study affinely-invariant geodetic models of such affine systems. It is shown that the dynamics of bounded elastic vibrations may be encoded in such geodetic models in the very form of the kinetic energy expression. Some special solutions like the relative equilibria are discussed. We start with the general approach to group-theoretical degrees of freedom and then discuss peculiarities of the affine group and certain other groups underlying collective dynamics.

1. Introduction

It is well-known that one is faced with rather serious analytical difficulties when dealing with complex systems, in particular multi-particle ones. In general there is no hope for analytical solutions and any qualitative or approximate analysis. Fortunately, quite often degrees of freedom of such systems are in a sense hierarchically ordered in such a way that a relatively small part of them is approximately decoupled from the remaining ones and ruled by approximately autonomous dynamics. And this dynamics gives an account of the main features of the object, relevant for the considered phenomena. Such hierarchy of degrees of freedom usually appears due to some peculiarities of intermolecular forces and quite often it has to do with geometry of the physical space or some other spaces relevant for the problem. The leading parameters deciding about the main dynamical features of the system are usually referred to as collective modes. The rules of the collective dynamics are either (more or less qualitatively) derived from the micromodel or somehow guessed on the basis of certain natural symmetry demands. The very